A STUDY OF THE EFFECT OF NON-NORMAL DISTRIBUTIONS UPON SIMPLE LINEAR REGRESSION

by

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TABLE OF CONTENTS

INTRODUCTION

Background

This paper presents a study of the effects upon the simple linear regression equation for data which is treated as if it came from a normal distribution when in fact it did not. To facilitate this discussion, it is necessary to first discuss a number of terms. The primary concept is the regression function which expresses one variable as a function of one or more other variables. This variable is called the dependent variable while the other variables are called the independent variables. Simple linear regression means that the function is of the first degree in the dependent and independent variables and that the function contains only one independent variable. Non-normal refers to any probability density function other than that of the normal distribution. In this paper only distributions of the gamma type are considered. Figure 1 illustrates a simple linear regression.

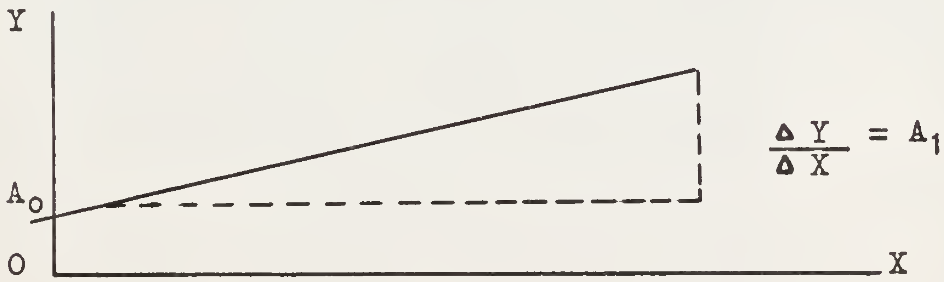

Fig. 1. A simple linear regression of the form $Y = A_0 + A_1 X$.

Since very little is known about the effects of non-normal

distributions on the normal regression equations, the simplest case, that is, the simple linear regression function was examined. This approach serves to determine the necessity of the investigation of the problem of non-normal regression and the value of examining more complex cases. Consequently, a brief survey of other classes of regression functions will outline other opportunities for research.

In recognizing that estimates of parameters from random samples will vary with each sample, and thus deviations of estimated parameters from true parameters will occur, it is necessary to state the simple linear regression function as

$$E(Y|X) = A_o + A_1 X \tag{1}$$

where E denotes the expected value of the dependent variable Y given a value of the independent variable X and where A_o and A_1 are the regression coefficients. In addition to this simple regression other classes of regressions are: multiple linear regression (more than one independent variable) which is illustrated in Fig. 2 of Plate I; simple non-linear regression (one independent variable of a degree greater than one) which is shown in Fig. 3 of Plate I; and multiple non-linear regression (more than one independent variable with at least one of these being of a degree greater than one) as shown in Fig. 4 of Plate I. These regression functions are stated as follows:

$$E(Y|X_1, X_2, \ldots X_n) = A_o + A_1 X_1 + A_2 X_2 + \ldots + A_n X_n \tag{2}$$

for multiple linear regressions;

EXPLANATION OF PLATE I

Fig. 2. Multiple linear regression of the form

$$E(Y|X_1, X_2) = A_o + A_1X_1 + A_2X_2.$$

Fig. 3. Simple non-linear regression of the form

$$E(Y|X) = A_o + A_1X + A_2X^2 + A_3X^3.$$

Fig. 4. Multiple non-linear regression of the form

$$E(Y|X_1, X_2) = A_o + A_1X_1 + A_2X_1^2 + A_3X_1^3 + B_1X_2 + B_2X_2^2 + B_3X_2^3.$$

PLATE I

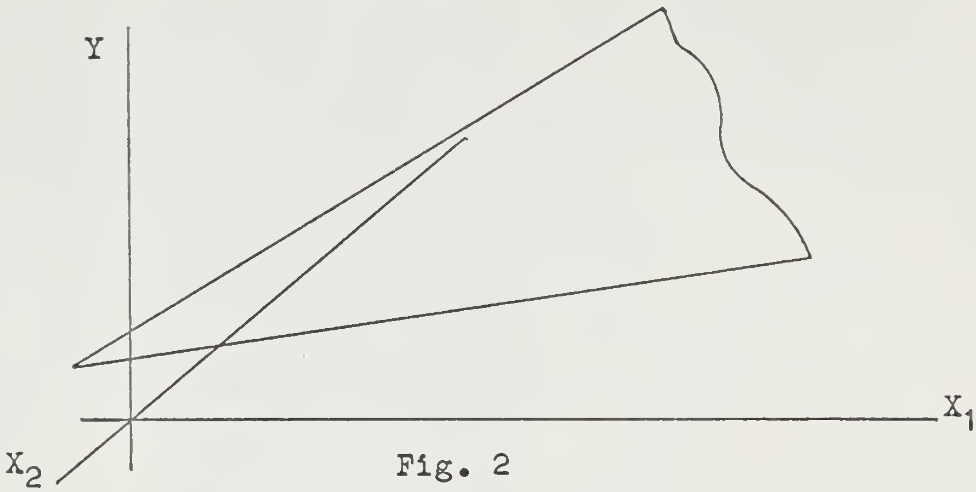

Fig. 2

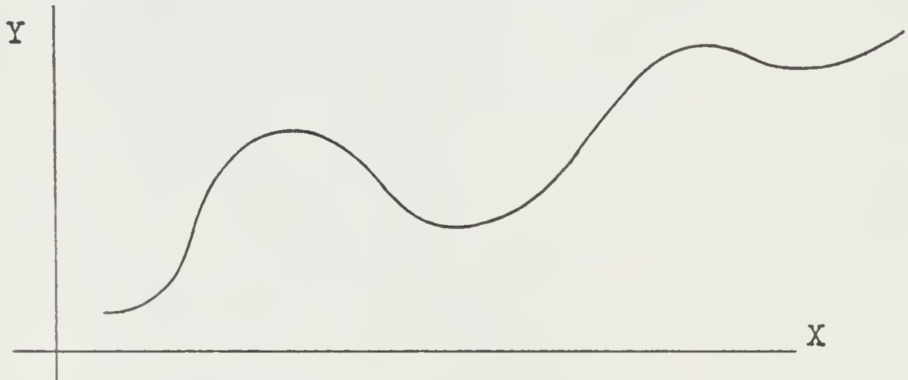

Fig. 3

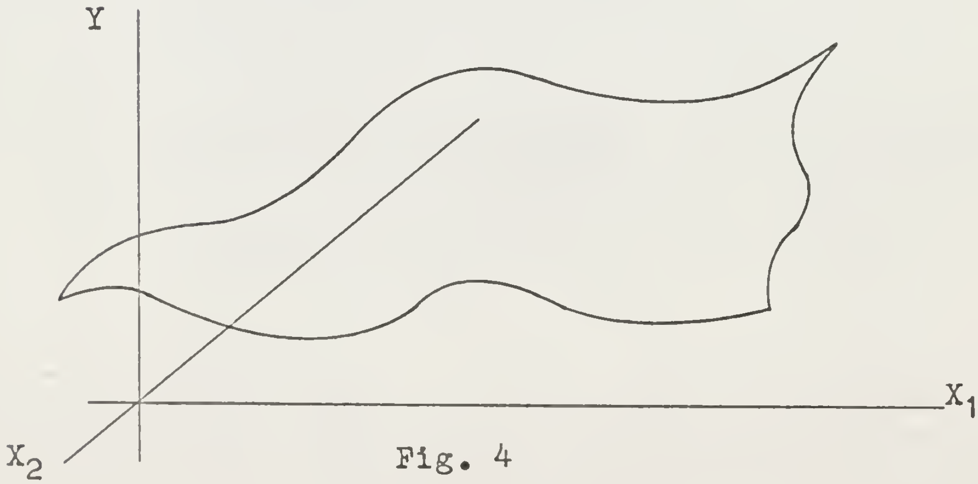

Fig. 4

$$E(Y|X) = A_o + A_1X + A_2X^2 + \ldots + A_nX^n \tag{3}$$

for the simple non-linear type of regressions; and

$$\begin{aligned} E(Y|X_1, X_2, \ldots X_n) = A_o &+ A_1X_1 + A_2X_1^2 + \ldots + A_iX_1^i + \ldots \\ &+ B_1X_2 + B_2X_2^2 + \ldots + B_jX_2^j + \ldots \\ &+ C_1X_n + C_2X_n^2 + \ldots + C_kX_n^k \end{aligned} \tag{4}$$

for the multiple non-linear type of regression.

Problem

Theoretically, there are two approaches to regression analysis. These two approaches are described as Model I and Model II by Snedecor (20). The Model I class of regression functions is discussed in this paper. The basic difference between these two models is the way in which the values of the independent variables are interpreted. In the case of simple regression and under the conditions of Model I, the dependent variable Y is assumed to be normally distributed about the regression line while the independent variable X is not treated as a variable but as an observable parameter. Model II defines the dependent variable Y and the independent variable X as being jointly distributed as normal variates, that is, having a bivariate normal distribution. This distinction is illustrated in Fig. 5 and Fig. 6 of Plate II. Consequently, Model I may be applied whenever the variation in the independent variable is due only to error in measuring. In addition, Model I is simpler in its assumption, since it requires knowing only the conditional probability density function,

EXPLANATION OF PLATE II

Fig. 5. Model I of regression analysis.

Fig. 6. Model II of regression analysis.

PLATE II

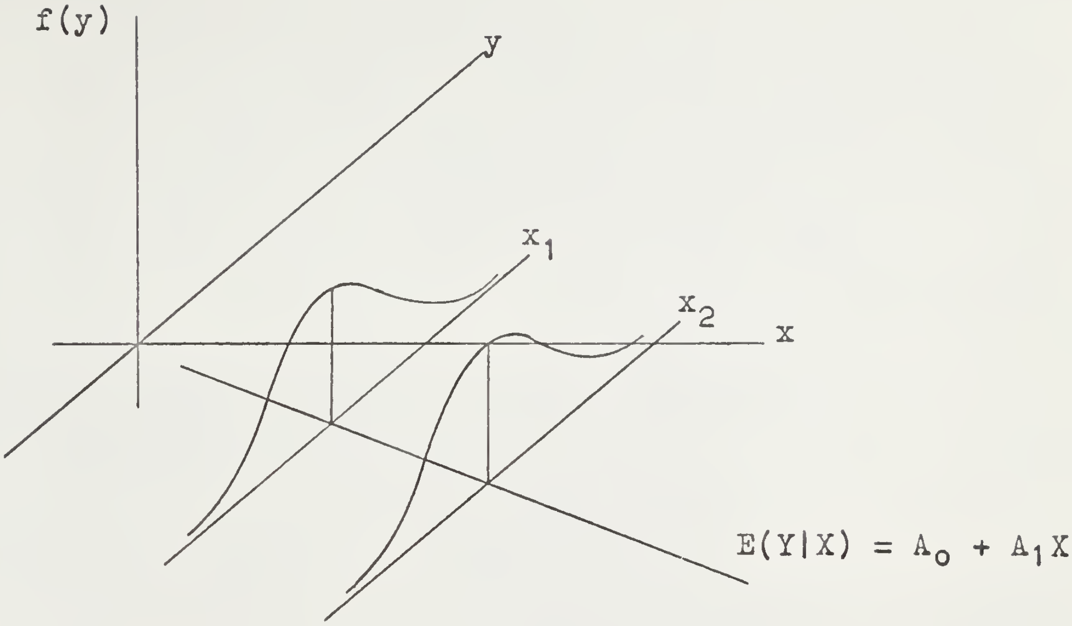

Fig. 5

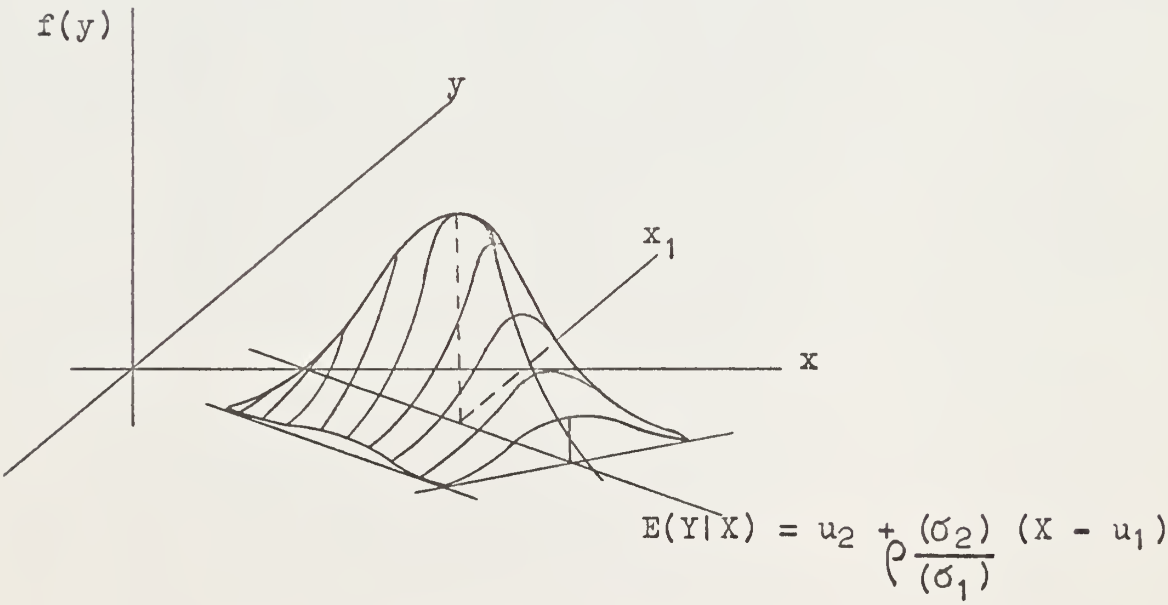

Fig. 6

$f(x|y)$, whereas Model II requires a knowledge of the joint probability density function. Model I and Model II are equivalent for purposes of computations, since their regression equations

$$E(Y|X) = A_0 + A_1X \tag{5}$$

and

$$E(Y|X) = u_2 + \rho(\sigma_2/\sigma_1)(X - u_1) \tag{6}$$

are equivalent for variables that have a normal distribution. In equation (6) u_1 and σ_1 are the population parameters for the independent variable, u_2 and σ_2 are the parameters for the dependent variable, and ρ is the correlation coefficient. Thus, since Model I has less stringent assumptions which can more easily be satisfied theoretically, the investigation is confined to Model I.

If in both models the variables are assumed to be normally distributed, then Model I assumes a univariate normal distribution and Model II assumes a conditional distribution. For Model I, this is expressed as

$$Y = A_0 + A_1X + \epsilon \tag{7}$$

where Y is a specific value of the dependent variable, A_0 and A_1 are the population parameters, and ϵ is a random variable from a normal distribution with mean equal to zero and variance equal to σ^2.

For a variable with a normal distribution, Model I has the following conditional density function

$$f(y|x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left[\frac{Y - (A_0 + A_1 X)}{\sigma}\right]^2} \tag{8}$$

where the parameters A_o, A_1, and σ are population parameters. The joint distribution of random samples of the random variables $X_1, X_2, \ldots X_n$, each with the probability density function f(x), has the following density function

$$f(x_1, x_2, \ldots x_n) = f(x_1) \cdot f(x_2) \cdots f(x_n) \tag{9}$$

for independent sample observations.

With this joint density function it is possible to find estimates of these parameters as functions of the sample observations. One technique for doing this is the method of "maximum likelihood" which is used here to obtain estimates of A_o, A_1, and σ.

> In principle, the method of "maximum likelihood" selects that value of a parameter θ for which the probability (or probability density) of attaining a given set of sample values is a maximum.[1]

This function is called the likelihood function. The method of maximum likelihood estimation consists of finding the maximum of the likelihood function which can usually be done by taking derivatives equal to zero. The likelihood function for Model I, denoted as L, is expressed as follows

$$L = \frac{1}{(\sqrt{2\pi}\,\sigma)^n} e^{-\frac{1}{2}\sum_{i=1}^{n} \left[Y_i - (A_o + A_1 X_i)\right]^2}. \tag{10}$$

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[1] J. E. Freund, Mathematical Statistics, pp. 223-224.

It is noted in this model that finding the maximum of the logarithm of the likelihood function is equivalent to finding the maximum of the likelihood function, thus

$$\ln L = -\,n \ln \sigma - \frac{n}{2} \ln 2 - \frac{1}{2\sigma^2} \sum_{i=1}^{n} \left[Y_i - (A_o + A_1 X_i)\right]^2. \tag{11}$$

Taking the partial derivatives with respect to A_o, A_1, and σ respectively and equating each of these derivatives to zero, the following is obtained

$$\frac{\partial \ln L}{\partial A_o} = \frac{1}{\sigma^2} \sum_{i=1}^{n} \left[Y_i - (A_o + A_1 X_i)\right] = 0 \tag{12}$$

$$\frac{\partial \ln L}{\partial A_1} = \frac{1}{\sigma^2} \sum_{i=1}^{n} \left[Y_i - (A_o + A_1 X_i)\right] = 0 \tag{13}$$

$$\frac{\partial \ln L}{\partial \sigma} = -\,\frac{n}{\sigma} + \frac{1}{\sigma^3} \sum_{i=1}^{n} \left[Y_i - (A_o + A_1 X_i)\right]^2 = 0 \tag{14}$$

of which equations (12) and (13) are commonly called the <u>normal</u> equations. Simultaneous solution of these equations provides the following maximum likelihood estimates of A_o, A_1, and σ:

$$\hat{A}_o = \frac{(\sum_{i=1}^{n} X_i^2)(\sum_{i=1}^{n} Y_i) - (\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} X_i Y_i)}{n\,(\sum_{i=1}^{n} X_i^2) - (\sum_{i=1}^{n} X_i)^2}, \tag{15}$$

$$\hat{A}_1 = \frac{n\,(\sum_{i=1}^{n} X_i Y_i) - (\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{n\,(\sum_{i=1}^{n} X_i^2) - (\sum_{i=1}^{n} X_i)^2}, \tag{16}$$

$$\hat{\sigma} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left[Y_i - (A_o + A_1 X_i)\right]^2} \tag{17}$$

The above development of regression theory is the usual treatment in regression analysis. A search of the literature indicates that very little has been done in applying regression analysis to non-normal distributions. Consequently, the purpose of this study has been to investigate the effect of certain non-normal distributions upon a regression function for the case where the independent variable X is an observable parameter (Model I). The regression function thus obtained was compared with the regression function that would be obtained if this same data was treated as though it had come from a normal distribution. The non-normal distributions investigated were special cases of the gamma distribution--exponential, chi-square, and negatively skewed normal distributions.

The gamma distribution has the probability density function

$$f(y) = \frac{y^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e^{-y/\beta} \tag{18}$$

where α is the shape parameter and β is the scale parameter. The mean of the distribution is equal to $\alpha\beta$ and the variance is equal to $\alpha\beta^2$. Setting $\alpha = 1$, and since $\Gamma(\alpha) = (\alpha - 1)!$ where α is an integer, the gamma distribution reduces to

$$f(y) = \frac{1}{\beta} e^{-y/\beta} \tag{19}$$

which is the well known exponential distribution with mean equal to β and variance equal to β^2. The conditional probability

density function for the case of simple linear regression is

$$f(y|x) = \frac{1}{A_o + A_1x} e^{-y/A_o + A_1x}. \tag{20}$$

Using the exponential distribution as an example, the concept of non-normal regression is illustrated in Fig. 7 and Fig. 8 of Plate III. Consequently, the problem is one of determining whether or not the linear regression function of

$$Y = A_o + A_1X \tag{21}$$

is the same for a given sample set of X's and corresponding Y's when they are treated as normally distributed variables when in fact they are non-normally distributed. The regression functions will be different if either or both of the regression coefficients are significantly different for the two distributions. Therefore, the problem can be resolved by testing for a significant difference between the regression coefficients, that is, by testing the hypotheses:

H_o: A_o(normal) = A_o(non-normal)

H_1: A_o(normal) $\neq$ A_o(non-normal)

H_o: A_1(normal) = A_1(non-normal)

H_1: A_1(normal) $\neq$ A_1(non-normal).

In the above, H_o is the null hypothesis which signifies no difference and H_1 the alternative hypothesis which indicates a significant difference between these regression coefficients.

EXPLANATION OF PLATE III

Fig. 7. Variate y normally distributed about a linear regression function.

Fig. 8. Variate y exponentially distributed about a linear regression function.

PLATE III

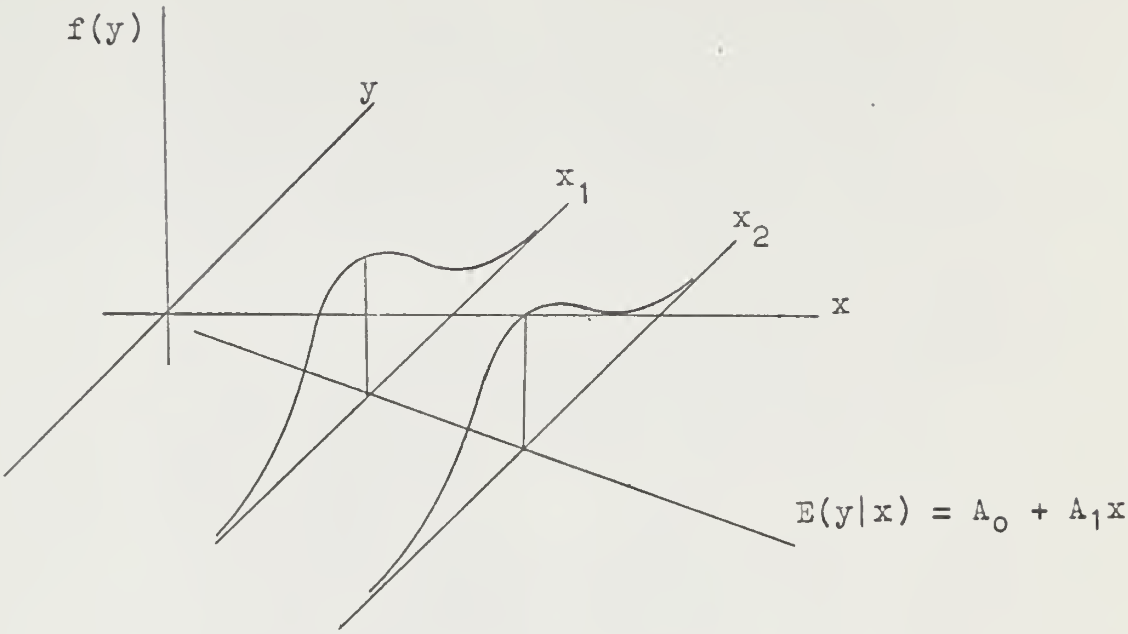

Fig. 7

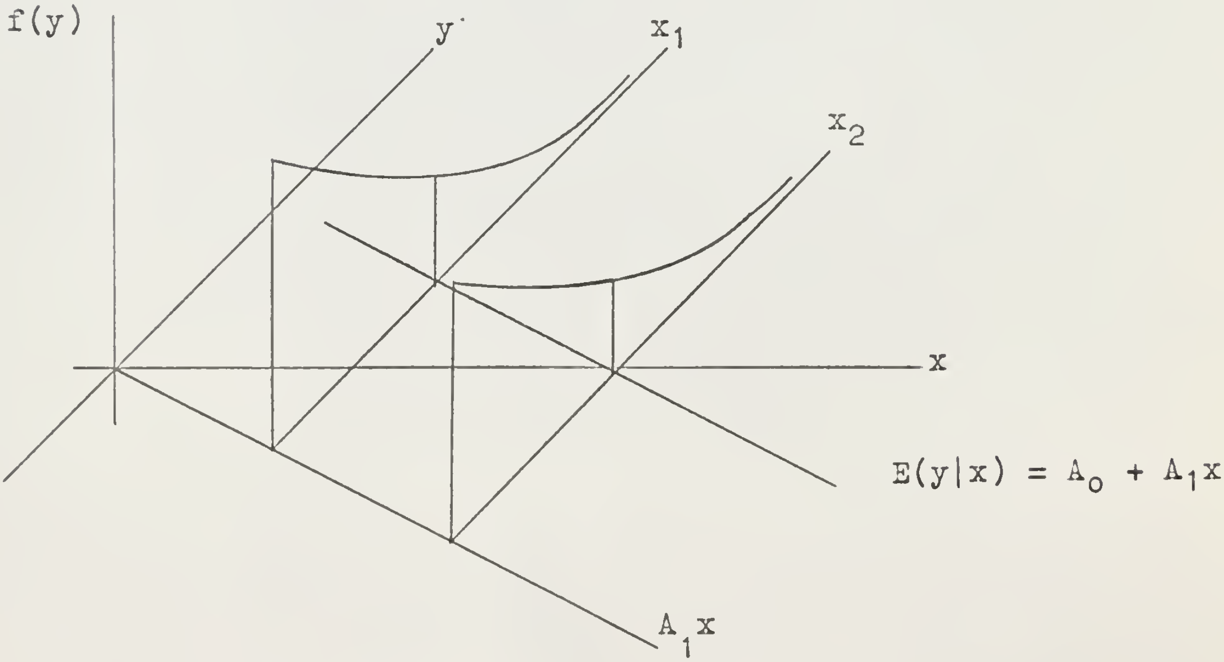

Fig. 8

REVIEW OF THE LITERATURE

As was previously indicated, the literature supporting research in the area of non-normal regression analysis is very sparse. Several works which are concerned with the theory of regression under the titles of certain non-normal distributions will be dealt with here; however, it will be shown that, for the most part, these studies employ the concept in a sense different from that suggested by the present study. It is common to find the terms "exponential regression" or "Poisson regression" in the existing literature, but most often these terms refer to the shape of the regression function rather than to the shape of the distribution of errors from the regression function. Consequently, these types of regressions can be more accurately described as special cases of the general class of non-linear regressions which was previously defined. Examples of these follow.

Studies which have been made of non-normal regression as it has been defined above consider Snedecor's (20) Model II of the population sampled, i.e., the case of a single independent variable, with both the dependent and independent variable distributed according to a specified probability density function. Conversely, the present work considers Snedecor's (20) Model I, i.e., the probability density function of the dependent variable being specified but with the independent variable assumed to be fixed as a set of observable parameters. Since Model I avoids the complexity of assuming a bivariate distribution, it has a

greater utility even though, computationally, the two models are treated similarly. The common use of Model I in regression analysis suggests that its properties be investigated in order to determine the limits within which this model can be utilized in practice.

Variations of Normal Regression

Villars (23) deals with what he terms exponential regression. He does examine Model I of regression analysis but instead of describing a linear regression function, he treats a regression function which is exponential in form. This analysis belongs to the first class of variations given in the introduction to this section. As in the maximum likelihood estimation of parameters, the conditional probability density function is still of the normal form. Thus

$$f(y|x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y - a + be^{-kx})^2}{2\sigma^2}} \qquad (22)$$

or

$$E(y|x) = a - be^{-kx} \qquad (23)$$

The errors from this "exponential" regression function are still assumed to be normally distributed. Villars is primarily concerned with forming estimates of the constants a, b, and k and with establishing their significance. He does succeed in fitting the function to a set of data representing a physical property of latex during a chemical process and in determining the significance of the fit.

Lipton and McGilchrist (15) offer a brief survey of

Richards' (19) and Steven's (21) asymptotic regression analyses, and they combine these techniques to arrive at maximum likelihood estimates of the parameters in a double exponential regression. Again the phrase "double exponential regression" is used in the sense that the regression function itself is an exponential curve. In this instance the function is of the form

$$E(Y|X) = \alpha - \beta_1\rho_1^2 - \beta_2\rho_2^2. \tag{24}$$

Again the distribution of Y about the regression function is assumed to be normally distributed. Therefore the problem can be reduced to one of non-linear (double exponential) normal regression rather than one of non-normal regression. Their analysis is devoted to making estimates of the parameters α, β_1, and β_2. Their technique is derived by using a combination of Steven's information matrix and Richards' method for selecting initial values. Two illustrations are provided--one in which convergence is obtained and one in which there is no convergence within the constrainted region. There is no consideration made of the effect of a non-normal distribution of Y.

An entirely different aspect of non-normal regression has been considered by Box and Watson (2) in which the effect of the non-normal distribution of Y upon the F-distribution is examined. in this case the distribution under consideration is that of the ratio of the "regression" mean square to the "residual" mean square in the analysis of variance table. With the exception of Durbin's (6) study, this is the only occurrence of the phrase

"non-normal regression" discovered in this literature survey. In this situation, the distribution of a ratio of mean squares, of which the numerator is computed from a regression function, is considered to be non-normally distributed. The non-normal distribution is actually associated with the F-distribution, and Box and Watson's work is, therefore, closely related to the Norton (18) study which shall be described later. The principal object of their study was to demonstrate the marked effect which the numerical values of the regression parameters have in deciding sensitivity in the F-statistic to a non-normal distribution. Their conclusion was that it is possible to choose the regression variables so that a non-normal distribution of observed variables is without a significant effect on the distribution of the test statistic. This is in agreement with Norton's study which set the precedent for this and many other studies regarding the robustness of the F-statistic, but still their result provides no solution to the problem of non-normal regression.

Considerations of Non-normal Regression

Gumbel (9) in his consideration of the non-normal Model II (bivariate case) of regression analysis questioned the same point of theory which the present study does. The essential difference between Gumbel's study and this one is the choice of theoretical models. Gumbel considers two bivariate exponential functions. The probability density function of the first is given as

$$f(x,y) = e^{-x(1 + \delta y - y)}\left[(1 + \delta x)(1 + \delta y) - \delta\right] \tag{25}$$

where δ provides a measure of correlation between x and y, and the probability density function of the second is

$$f(x,y) = e^{-x-y}\left[1 + \alpha(2e^{-x} - 1)(2e^{-y} - 1)\right] \tag{26}$$

where the sign of α determines the sign of the correlation between x and y. Based upon the conditional density function, Gumbel determines the following regression functions

$$E(X|Y) = \frac{1 + \delta + \delta y}{(1 + \delta y)^2} \tag{27}$$

and

$$E(X|Y) = 1 + \frac{\alpha}{2} e^{-y} \tag{28}$$

respectively. The corresponding regression equation of X on Y in the case of the bivariate normal is given by

$$E(X|Y) = u_1 + \rho(\sigma_1/\sigma_2)(y - u_2) \tag{29}$$

which is seen to be linear, while the same functions for the bivariate exponential regression curves are quadratic and exponential, respectively. This result illustrates a noteworthy point. Since the present study indicated no significant difference between normal regression functions and corresponding exponential types of regression function for Model I, and since Gumbel's study shows a striking difference at least in the form of analagous regression functions for Model II, the distinction between Model I and Model II is not a trivial one, expecially when there is a departure from the conditions of normality.

This finding suggests a study similar to the present one in which Model II would be considered, for, although Gumbel suggests a significant difference in his study, he does not clearly demonstrate it.

Chernoff (4,5) dealt directly with the concept involved in the present study. He considers Model I of regression analysis--a fixed independent variable and a dependent variable varying according to a specified distribution. He also considers a non-normal distribution (exponential) of errors from the regression function. Two general types of exponential regression are investigated--a quadratic, non-linear exponential and a double exponential regression or an exponential, non-linear exponential regression. The basic conditional density function is of the form

$$f(y|x) = \theta e^{-\theta y} \tag{30}$$

where $\theta = \theta_1 x + \theta_2 x^2$ and $\theta = \theta_1 e^{-\theta_2 x}$ respectively.

Chernoff utilized Elfving's (7) optimal weighting for the least squares estimation of parameters which finds the smallest convex set containing both the parameters of the regression $(x_1, x_2, \ldots x_n)$ and their negatives $(-x_1, -x_2, \ldots -x_n)$. This set is therefore the weighted averages of such points. Using these estimates of regression parameters, Chernoff assumed values for the parameters and was able to compute optimal levels of stress in terms of the cost involved in accelerated life-testing. It is the technique of estimating these parameters which makes

Chernoff's work a pertinent reference; however, he used these estimates of the parameters to provide a unit cost in designing accelerated life-testing experimentation and did not consider the effect of a non-normal distribution of Y upon the regression function. Also, since only the quadratic and exponential non-linear cases of regression were considered, there is no means by which the estimates of regression coefficients can be tested for significant variation from the common linear estimates of regression coefficients as provided by the normal equations. Consequently, the Chernoff study offered very little in support of the present experiment.

Jorgenson (11) also considered Model I of regression analysis with a non-normal distribution of points about the regression function. He confined himself to the discrete Poisson distribution with a multiple regression. His conditional density function is of the form

$$f(u_i|\lambda_1, \lambda_2, \ldots \lambda_k) = \frac{\left(\sum_{j=1}^{k} \lambda_j T_{ij}\right)^{u_i}}{u_i!} e^{-\sum_{j=1}^{k} \lambda_j T_{ij}}. \tag{31}$$

Here the method of maximum likelihood provides the following estimate of $\hat{\lambda}_j$.

$$\sum_{i=1}^{m} T_{ij} = \sum_{i=1}^{m} \frac{u_i T_j}{\sum_{j=1}^{k} \hat{\lambda}_j T_{ij}} \tag{32}$$

where $\hat{\lambda}_j$ is the estimate.

This equation must be solved for $\hat{\lambda}_j$ through an iterative process. Jorgenson proceeded to compute estimates through two

alternative iterative techniques based upon a set of data. He demonstrated that the first iteration of each technique has the same large sample properties as the maximum likelihood estimator itself. He then provided three alternate estimators of the regression parameters which involved much less computation than either of the two iterative techniques used in determining the maximum likelihood estimator and which provided a good degree of accuracy. Jorgenson's purpose was only to form various estimates of the parameters in a Poisson regression. He made no attempt to relate his estimations to the normal-theory assumptions of regression analysis. Estimates of regression coefficients based upon a distribution other than the normal distribution can be justified only if they can be shown to be significantly different than the estimates based on the normal distribution or if they provide a computational facility. Since Jorgenson's involved iterative techniques do not offer this facility, his study is at best incomplete.

Finally, Durbin (6) made reference to non-normal regression in his study of the relative efficiencies of maximum likelihood estimation and least squares estimation of parameters in time-series regression models. He did so in the proper sense of the term; however, he was content to demonstrate mathematically that the method of least squares provides optimum estimates only when the errors from the regression function are normally distributed. Under the conditions of non-normal error, the method of maximum likelihood provides a more efficient estimate,

although the method of least squares estimation approaches these estimates as a limiting function and may therefore be used for all practical purposes.

Durbin's major conclusions concern the lagging and leading of variables in time-series analysis. He found that the properties of the least square estimates are the same asymptotically as those of the least square coefficients of ordinary regression models containing no lagged variables. Consequently, he failed to consider differences between the regression functions as estimated through the method of maximum likelihood under conditions of a normal distribution and a non-normal distribution respectively.

A comparison of the references given above with the proposal made in the introduction of this paper will show that the basic question of non-normal regression remains unanswered. This is: Does the non-normal distribution of a variate Y about a regression function affect the parameters of that regression when the distribution of Y is assumed to be normal, that is, is there a significant difference in the true parameters and the parameters obtained by assuming the distribution of Y to be normal? Although this study does not provide a definitive answer to these questions, it does perhaps give an indication of what these answers are.

METHOD

The exponential distribution will be used as an example in the following discussion since its properties are the same as the other gamma distributions being considered. The other distributions considered are treated similarly. Repeating the method of maximum likelihood estimation and applying it to the previously derived conditional probability density of the exponential distribution (gamma distribution with $\alpha = 1$, $\beta = 1$) the following likelihood function is obtained

$$L = \frac{1}{\prod_{i=1}^{n} (A_o + A_1 X_i)} e^{-\sum_{i=1}^{n} Y_i/(A_o + A_1 X_i)} . \tag{33}$$

Taking logarithms the expression becomes

$$\ln L = -\sum_{i=1}^{n} \ln (A_o + A_1 X_i) - \sum_{i=1}^{n} Y_i/(A_o + A_1 X_i). \tag{34}$$

Finding the partial derivative with respect to A_o and A_1 and equating these to zero, the following system of equations is obtained

$$\frac{\partial \ln L}{\partial A_o} = -\sum_{i=1}^{n} \frac{1}{A_o + A_1 X_i} + \sum_{i=1}^{n} \frac{Y_i}{(A_o + A_1 X_i)^2} = 0. \tag{35}$$

$$\frac{\partial \ln L}{\partial A_1} = -\sum_{i=1}^{n} \frac{X_i}{A_o + A_1 X_i} + \sum_{i=1}^{n} \frac{X_i Y_i}{(A_o + A_1 X_i)^2} = 0. \tag{36}$$

Equations (35) and (36) are analagous to the normal equations (12) and (13) and will be called the non-normal equations

for purposes of contrasting the underlying assumptions.

The most direct way of determining a significant difference between the normal estimates of the regression coefficients and the non-normal estimates of these coefficients is to solve the non-normal equations. However, an inspection of equations (35) and (36) will show that they cannot be solved explicitly for A_0 and A_1. Therefore an alternative approach was sought. This resulted in selecting several representative models from the gamma family of distributions about known regression functions. Samples were then drawn from these known distributions and treated as normally distributed samples for the purpose of computing estimates of the regression equation. These estimates were then compared with the known regression coefficients.

This method has a well-established precedent in the Norton (18) and Bartlett (1) studies on the effect of non-normal distributions in the F-statistic and in the t-statistic respectively. The design of the experiment was closely patterned after the Norton experiment. Norton randomly selected samples arranged on cards from populations of size 10,000. A total of six populations were used. These populations were leptokurtic, rectangular, moderately positively skewed, markedly positively skewed, and J-shaped; in addition, a normal population was used as a "control group". From each of these populations, Norton selected 3,000 independent sample sets. The sets each consisted of k random samples of n cases each, and k and n were different for different F-distributions. Each population provided a different

treatment, and each drawing offered a different set of data. The ratio of the mean squares for the between-treatment variation to the within-treatment variation was computed, and a distribution of these ratios was tabulated for 3,000 trials. The percentage of these ratios exceeding specified values were recorded. In this way the discrepancies in the critical upper-tail region between the empirical distribution thus obtained and the normal theory F-distribution were readily described. Norton found that the discrepancies between tabulated probabilities for the F-distribution and the empirical probabilities were very small. Therefore, he concluded that the F-statistic was robust or generally insensitive to the distribution of the treatment data.

Similarly a selected number of gamma distributions were generated in this study by means of the IBM 1620 computer. The gamma probability density function of

$$f(y) = \frac{y^{\alpha - 1}}{\beta\Gamma(\alpha)} e^{-y/\beta} \tag{37}$$

was used for values of the shape parameter α equal to 1, 2, and 4 and with the scale parameter β equal to unity. Thus the following probability density functions were used where the parameters are:

$\alpha = 1, \quad \beta = 1$, and

$$f(y) = e^{-y} \tag{38}$$

which is an exponential distribution with $u = \alpha\beta = 1$ and $\sigma^2 = \alpha\beta^2 = 1$.

$\alpha = 2, \quad \beta = 1$, and

$$f(y) = ye^{-y} \tag{39}$$

which is approximately a chi-square distribution with $u = \alpha\beta = 2$ and $\sigma^2 = \alpha\beta^2 = 2$.

$\alpha = 4$, $\beta = 1$, and

$$f(y) = \frac{y^3}{6} e^{-y} \tag{40}$$

which is a moderately negatively skewed normal distribution with $u = \alpha\beta = 4$ and $\sigma^2 = \alpha\beta^2 = 4$. These distributions are illustrated in Fig. 9, Fig. 10, and Fig. 11 of Plate IV.

The gamma distribution was chosen because it represents a class of distributions which can readily be derived by selecting appropriate values for its parameter. The gamma distributions given above were chosen because of their common use in statistical analysis, and because they offer a sequence of decreasing skewness. This sequence would provide an opportunity for detecting the degree of skewness at which significant variation occurred in the estimation of regression coefficients.

These distributions correspond to the skewed distributions which Norton used. A set of normal distributions was also generated and used as a control for the experiment. An associated normal distribution was used as a control for each gamma distribution in order to have a normal distribution with a variance equal to each of the experimental distributions. Thus three normal distributions served as controlling distributions: N(1,1); N(2,2); and N(4,4). In this way the effect of variance, if any, upon the estimation of the regression parameter could be observed.

EXPLANATION OF PLATE IV

Fig. 9. Exponential distribution with $u = 1$ and $\sigma^2 = 1$.

Fig. 10. Chi-square distribution with $u = 2$ and $\sigma^2 = 2$.

Fig. 11. Moderately skewed normal with $u = 4$ and $\sigma^2 = 4$.

PLATE IV

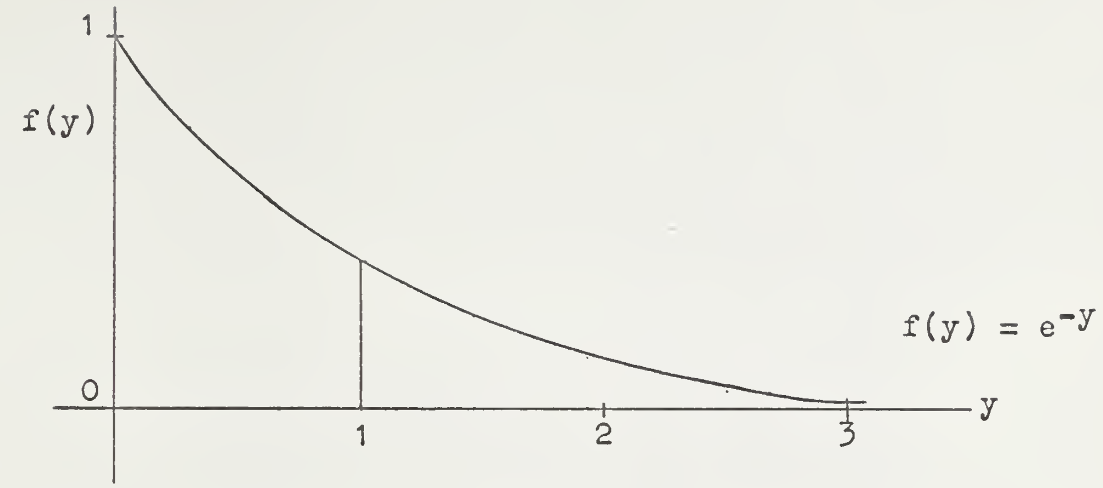

Fig. 9

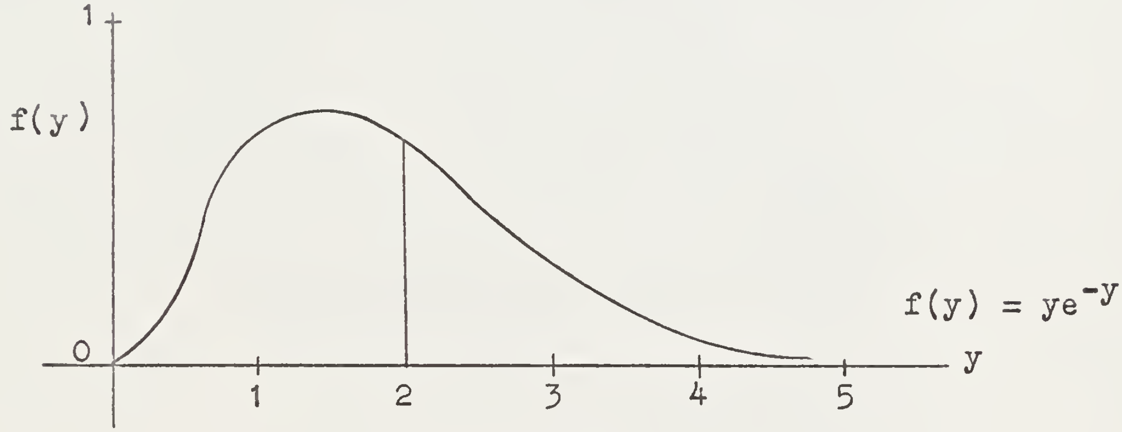

Fig. 10

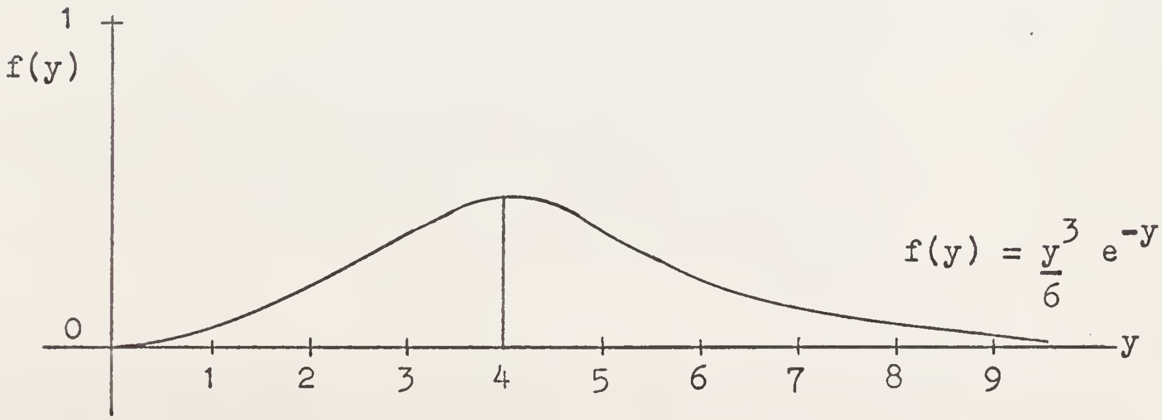

Fig. 11

Generating Non-normal Random Variables

The scheme used to generate the variables from these distributions is a simple one which is based upon a theorem stated by Hogg and Craig (10).

> Let X be a random variable of the continuous type having a probability density function f(x) and distribution function F(x). Then the random variable Z = F(x) has a uniform distribution with probability density functions
>
> $$h(Z) = 1, \qquad 0 \leq Z \leq 1$$
>
> $$h(Z) = 0, \qquad \text{elsewhere.}$$[2]

This technique may be illustrated in the case of the gamma distribution with $\alpha = \beta = 1$ which is the exponential distribution

$$f(y) = \frac{1}{\beta} e^{-y/\beta} \qquad 0 \leq y \leq \infty \tag{41}$$

and

$$F(y) = 1 - e^{-y/\beta}, \tag{42}$$

thus

$$Z = 1 - e^{-y/\beta} \tag{43}$$

or

$$Y = - \beta \ln (1 - Z). \tag{44}$$

Therefore, when β has been fixed by assuming a regression function and Z, a random uniform variate, has been generated, the desired variate y is obtained by means of a simple compu-

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[2] R. V. Hogg, and A. T. Craig, Introduction to Mathematical Statistics, p. 178.

tation. This technique is readily adapted to any distribution for which the distribution function can be described.

A uniform random variate can be generated on a computer in several ways (13,16,22). Most of the available schemes use modular arithmetic as their basis for computation. The method selected here was suggested by Lehmer (13) and is a very efficient one.

> The scheme is this: Start with any ten-digit number of the form XYZ0000001 and call it R_0. Thereafter, $R_n = K \cdot R_{n-1} \pmod{10^{10}}$, where K is a fixed multiplier, which should be a ten-digit odd power of a prime that is relatively prime to 10... The recursion relation expressed above is interpeted as: to get the next number in the sequence, multiply the previous number by K. The result will be a 20-digit number. Select the right-hand ten digits of the product as the next number in the sequence. Treat the number as a ten-digit decimal.[3]

The value of K used to generate the sequence of numbers used in the experiment was $7^{11} = 1,977,326,743$. The scheme has the advantages that it is easy to perform on a computer such as the IBM 1620 used in this experiment. Also approximately 50 million numbers can be generated before the sequence repeats itself (13).

The sequence of numbers which was used in the experiment was tested for goodness-of-fit to the uniform distribution and for serial correlation by means of the Kendall-Babington-Smith (12) chi-square tests. The statistic used to test the goodness-of-fit was:

$$\chi_1^2 = \frac{k}{n} \sum_{i=1}^{k} (f_i - \frac{n}{k})^2 \tag{45}$$

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[3]R. G. Brown, Statistical Forecasting for Inventory Control, pp. 164-165.

with k-1 degrees of freedom, and the statistic for determining the significance of serial correlation was:

$$\chi_2^2 = \frac{k^2}{n}\sum_{i=1}^{k} (f_{i,j} - \frac{n}{k})^2 \tag{46}$$

with k^2-1 degrees of freedom and where n is the length of the sequence (equal to 500 in this case), k is the number of intervals (taken to be 10 in this instance), f_i is the frequency of the i-th interval, and $f_{i,j}$ is the frequency of a number in the i-th interval followed by a number in the j-th interval. The computer program for computing these statistics may be found in the appendix (p. 65) together with the results (p. 78) and a sample of the random number sequences (p. 82). The computed χ_1^2 statistic for the sequence of 500 numbers used in this experiment was 3.960 to be compared with a critical value of 16.92 with 9 degrees of freedom at a 0.05 level of significance. Similarily, the computed statistic χ_2^2 was 103.800 which is less than the critical value of 123.26 with 99 degrees of freedom and at a 5% level of risk. In addition the statistic, $\chi_2^2 - \chi_1^2$ (103.800 - 3.960 = 99.840) has asymptotically a χ^2 distribution with 90 degrees of freedom. The 5% critical value is 113.14. Therefore the random number generator which was used resulted in non-significant variation from a random uniform distribution with an associated probability of a Type I error of less than 0.05.

Generating Normal Random Variables

The same sequence of random numbers was used to generate the gamma distributions and the corresponding normal control distri-

butions; however, the method for generating the normally distributed variables differed slightly from that of the gamma distributions. Since the distribution function of the normal distribution is not an integrable form,

$$F(y) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{y} e^{-\frac{1}{2}\left[\frac{y - u}{\sigma}\right]^2} dy \tag{47}$$

Y cannot be found directly in terms of a random uniform variate. Although several approximations for evaluating this integral are available, an alternate method was used to generate random normal samples.

The moments of the uniform distribution are

$$u = \frac{\alpha + \beta}{2} \tag{48}$$

and

$$\sigma^2 = \frac{(\alpha - \beta)^2}{12} \tag{49}$$

where α and β are the endpoints of the interval, in this case, equal to 0 and 1 respectively. Therefore $u = \frac{1}{2}$ and $\sigma^2 = 1/12$. Therefore, the sum of twelve such variates gives a mean of $u = 12/2 = 6$ and a variance of $\sigma^2 = 12/12 = 1$. Subtracting 6 from the previous mean results in a mean of $u = 0$, while the variance $\sigma^2 = 1$. According to the Central Limit Theorem the resulting variable has an approximate standard normal distribution. The Central Limit Theorem states:

> If $x_1, x_2, \ldots,$ and x_n are independent random variates having the same distribution with the mean u, the variance σ^2, and the moment generating function $M_x(t)$, then if $n \longrightarrow \infty$ the limiting distribution of

$$Z = \frac{\bar{x} - u}{\sigma/\sqrt{n}}$$

is the standard normal distribution.[4]

Thus, since Z is distributed as N(0,1), and

$$Z = \frac{\bar{x} - u}{\sigma/\sqrt{n}} = \frac{\sum x_i - nu}{\sqrt{n}\ \sigma} \tag{50}$$

then $Y = \sum x_i$ is distributed as $N(nu, n\sigma^2)$. Consequently, the random variable $\omega = Y - 6$ is distributed as $N(nu - 6, n\sigma^2)$. Substituting the values of $n = 12$, $u = \frac{1}{2}$ and $\sigma^2 = \frac{1}{2}$, it is seen that Z is distributed as N(0,1). Thus a normal variate from any population can be approximated in this manner by specifying the parameters u and σ^2.

The sequence of standard normal variates generated by Lehmer's scheme for uniform random numbers was tested at the 5% level of significance by a chi-square goodness-of-fit test. Fourteen intervals were used, thereby providing thirteen degrees of freedom. The computed chi-square statistic was 11.123 which was compared with a critical value of 22.36. Therefore the sequence provides a standard normal distribution with more than a 95% level of confidence. The Fortran program used to compute this statistic can be found in the appendix (p. 67) together with the results (p. 81).

Distributed Random Samples

The conditional probability density function of the gamma

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[4]Freund, op. cit., p. 185.

distribution is

$$f(y|x) = \frac{(y - A_1x)^{\alpha - 1}}{\beta^{\alpha}\Gamma(\alpha)} e^{-(y - A_1x)/\beta} \tag{51}$$

where A_1x is the location parameter of the distribution. Integration by parts of this form yields the following distribution functions:

$$F(y|x) = 1 - e^{-(y - A_1x)} \tag{52}$$

for $\alpha = 1$ and $\beta = 1$ (exponential).

$$F(y|x) = 1 - (y - A_1x)\, e^{-(y - A_1x)} - e^{-(y - A_1x)} \tag{53}$$

for $\alpha = 2$ and $\beta = 1$ (approximately chi-square).

$$F(y|x) = 1 - \frac{(y - A_1x)^3}{6} e^{-(y - A_1x)} - \frac{(y - A_1x)^2}{2} e^{-(y - A_1x)} - (y - A_1x)\, e^{-(y - A_1x)} - e^{-(y - A_1x)} \tag{54}$$

for $\alpha = 4$ and $\beta = 1$ (moderately negatively skewed normal). The necessary derivations are found in the appendix (pp. 62-64).

Values were assumed for A_1 in the preceding functions; A_0 was set equal to the mean of the distribution at $X = 0$. These values of A_1 and A_0 are then the population parameters of the regression function. Random samples from the distributions were obtained by setting the distribution functions (52), (53), and (54) equal to random uniform variates for selected values of the observable parameter, X. Consequently, random samples of Y's were generated in this manner about the assumed regression

function, thus simulating the Model I regression. The regression coefficient or intercept A_o was set equal to the mean of each distribution, and +2, 0, and -2 were selected as the values of A_1. These sets of regression functions are illustrated in Fig. 12, Fig. 13, and Fig. 14 of Plate V and in Fig. 15, Fig. 16, and Fig. 17 of Plate VI. The values of A_1 were chosen to represent a class of positive, zero, and negative regression functions in the event that the slope of the function was a factor in obtaining a significant difference between estimates.

Solution for Sample Values by Iterative Techniques

It should be noted that of the three gamma distribution functions, only the first one, the exponential distribution function, can be solved explicitly for Y. This led to the use of an iterative process in solving the distribution function equations for the case of the chi-square and skewed normal distributions. Initially the Newton-Raphson method was used; this is of the form

$$Y_{n+1} = Y_n - \frac{f(Y_n)}{f'(Y_n)} . \tag{55}$$

For large values of Y, corresponding to cumulative probabilities of 0.90 or more, the ratio of the function to its derivative was great enough to cause an exponential overflow in the Fortran processor and thus this technique could not be used in this experiment.

EXPLANATION OF PLATE V

Fig. 12. Exponential regression functions used in the experiment.

Fig. 13. Chi-square regression functions used in the experiment.

Fig. 14. Skewed normal regression functions used in the experiment.

PLATE V

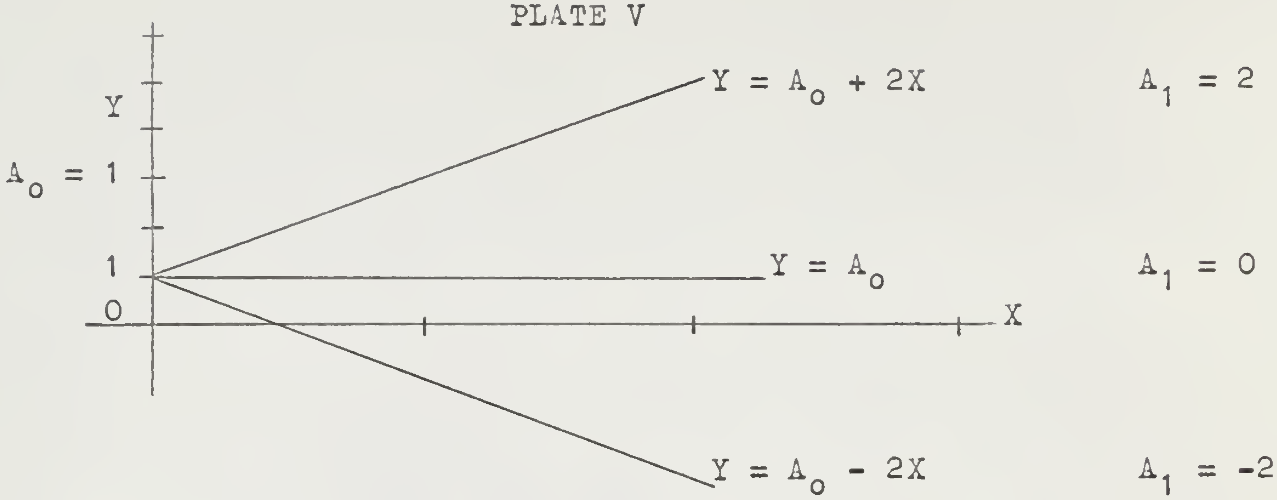

Fig. 12

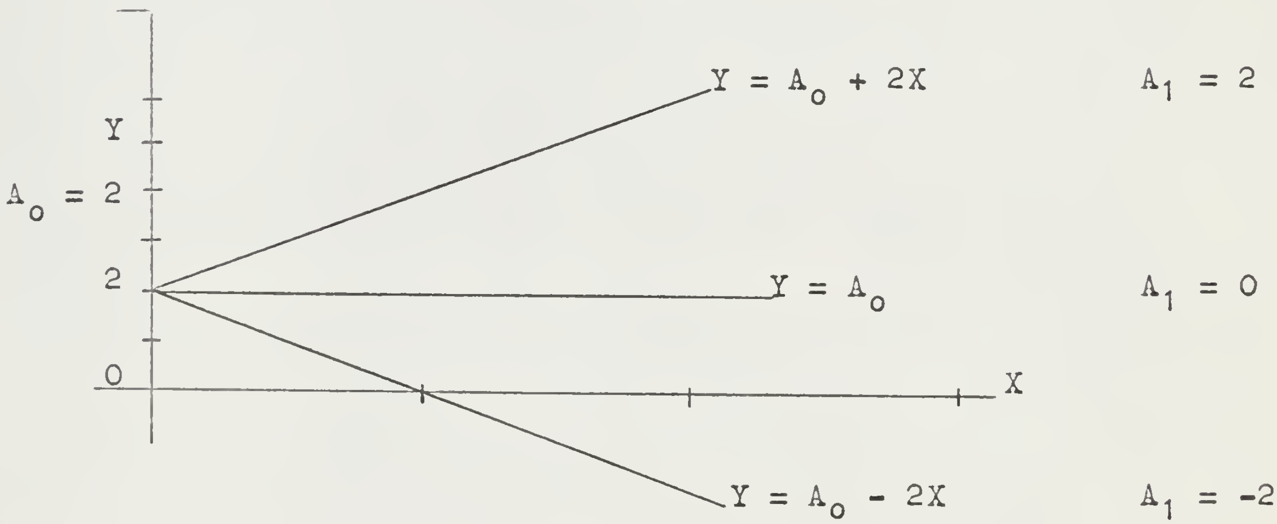

Fig. 13

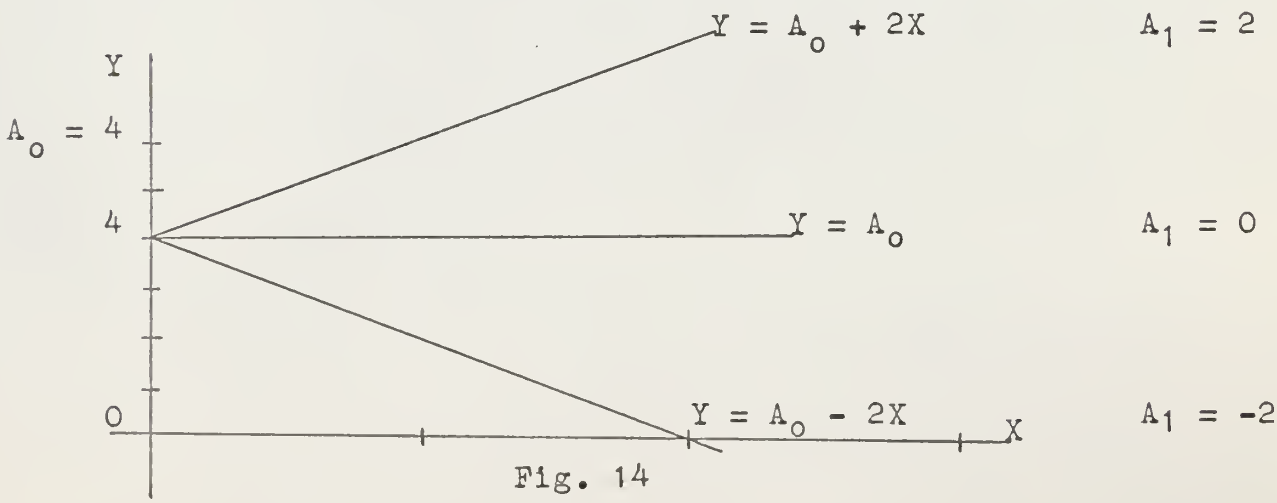

Fig. 14

EXPLANATION OF PLATE VI

Fig. 15. Normal regression functions with variance equal to 1.

Fig. 16. Normal regression functions with variance equal to 2.

Fig. 17. Normal regression functions with variance equal to 4.

PLATE VI

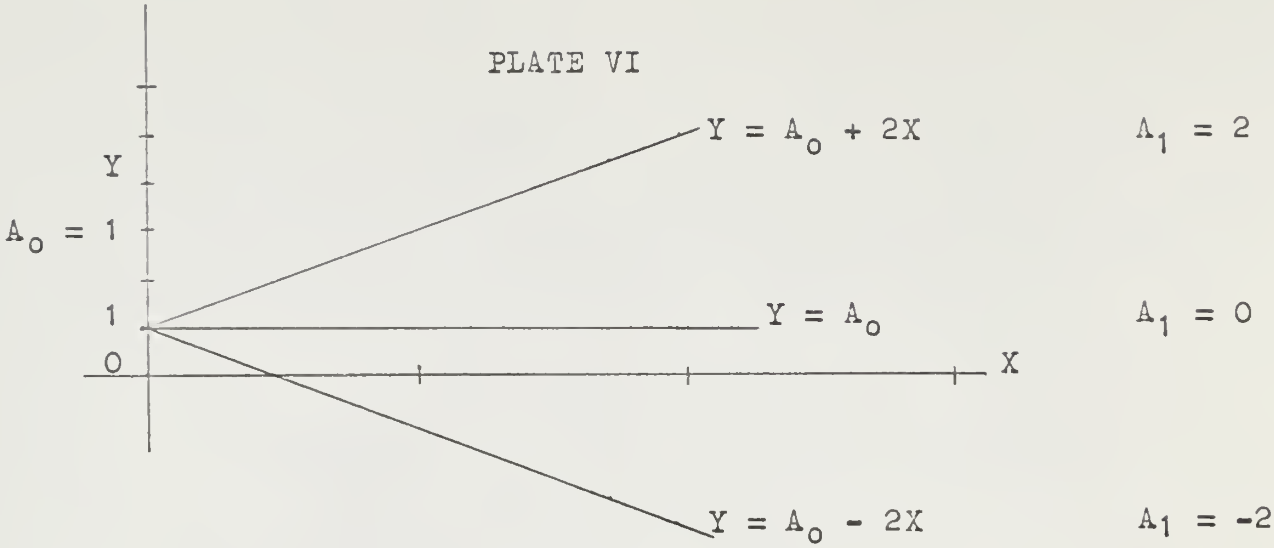

Fig. 15

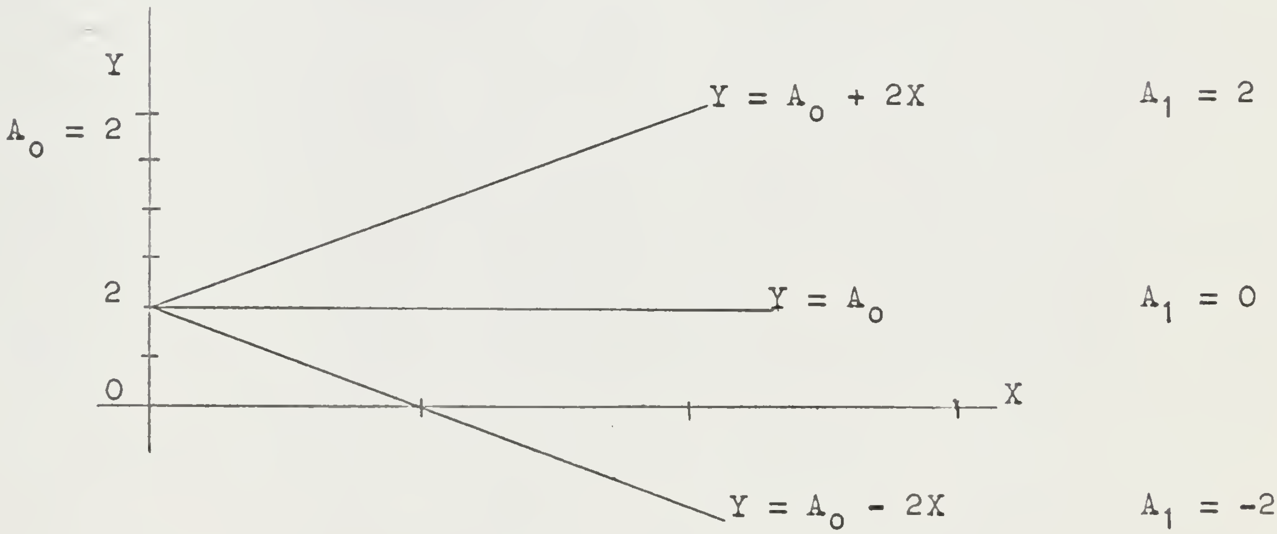

Fig. 16

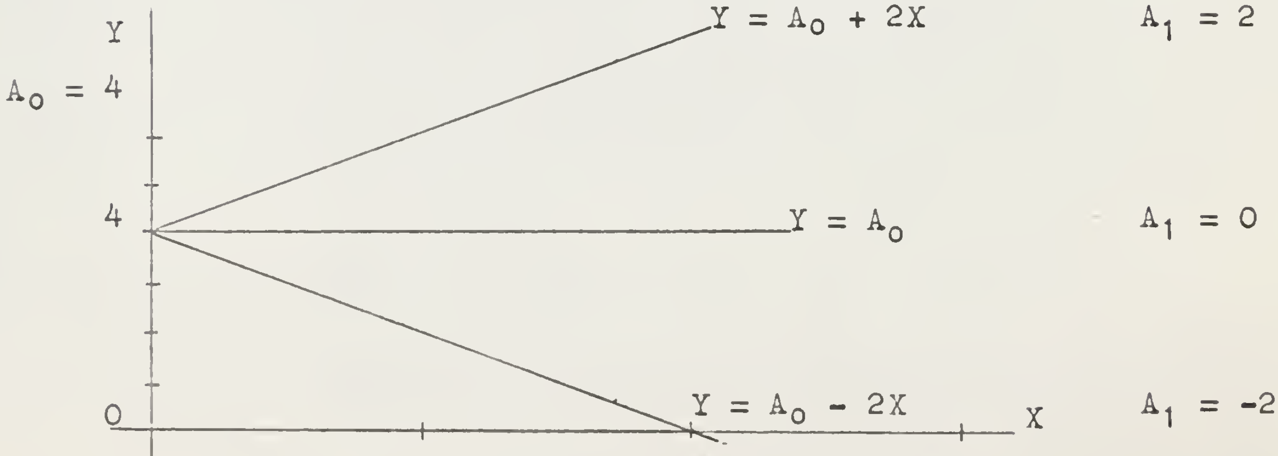

Fig. 17

The Bolzano iterative technique was also attempted. This method is typical of the straight-line interpolation techniques and it has the form

$$\gamma = \frac{\beta\,|\epsilon_\beta| + \alpha\,|\epsilon_\alpha|}{|\epsilon_\beta| + |\epsilon_\alpha|} \tag{56}$$

where α and β are points on either side of the solution and where ϵ_α and ϵ_β are the errors associated with these points. The next value for α or β in the process is γ depending upon the sign of the error resulting from the previous iteration. This routine did work, however, it was inefficient and was rejected since, as Y increased, the time required for the iterative process also increased to an excessive degree.

The method finally used was actually the simplest one available; this is Horner's method, commonly called the interval-halving technique. Horner's method provided a relatively rapid solution to the equations and the time required was roughly constant for any value of Y (25 seconds for the chi-square distribution and 45 seconds for the skewed normal distribution function). The process provided an error of absolute value less than 0.00001 between the generated uniform variate and the one computed by substituting successive values of Y. This precision is required because of the general shape of the gamma distribution function as shown in Fig. 18 below. In the range 10 to infinity, a very small error in Z will result in a very large error in Y. The time increase in the Bolzano iterative process was also due to the small slope of this curve above Y = 10.

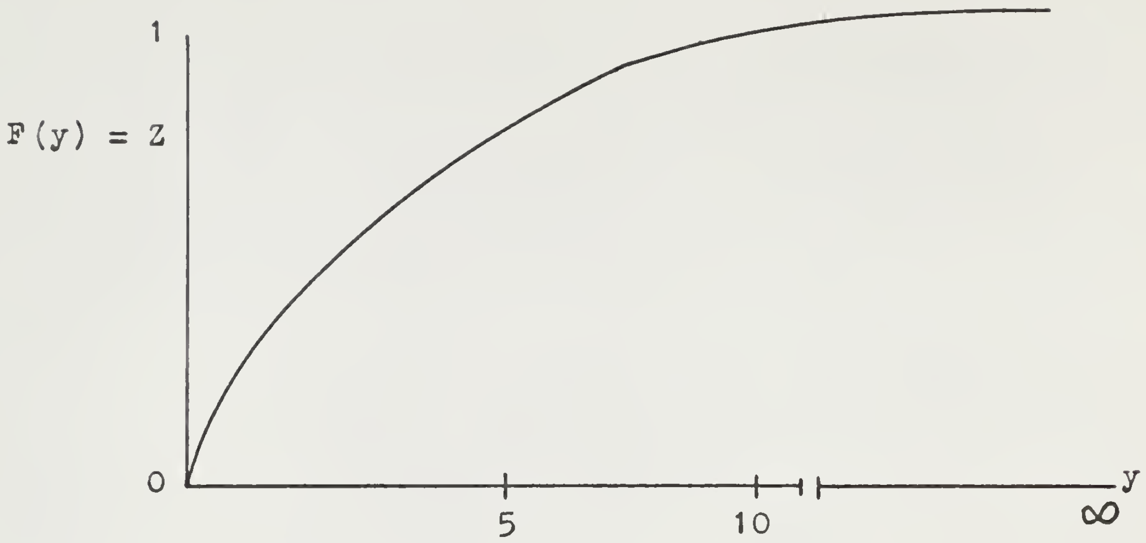

Fig. 18. General shape of the gamma distribution function.

The Experiment

A total of 1,800 points were generated in the experiment. Twenty points were generated about each of the regression functions shown in Plates V and VI; these were based on twenty values of the independent variable X. The sample size is arbitrary and was selected so as to give a degree of freedom large enough to provide fairly sensitive tests. The procedure was repeated five times for each regression function, using the same set of independent variables, resulting in a sample of 100 points about each of the eighteen regression functions. Three different sets of independent variables were used with one set being used for all positively sloped regressions, with another being used for all horizontal regression functions, and with the third being used for all negatively sloped regression functions. These X values were selected from Snedecor's (20) table of random

numbers and these are listed in Table 1. In this way, 90 sets of paired (X,Y) data were obtained, since each regression had five sets of data. These sets are found in the computer output of the appendix (pp. 83-124). Each set of data was used to compute estimates of the regression coefficients based upon the normal equations--equations (12) and (13) on p. 10. These estimates are found in the appendix (pp. 83-124) and are also tabulated in Table 2 as part of the results (p. 47). The five sets of data for each regression function were combined to make an overall estimate of the regression coefficients; these results are also found in the computer output of the appendix (pp. 125-161) and in Table 3 as part of the results (p. 52).

Test of Hypothesis

The hypotheses were tested using the following t-statistics (8) for:

A_o

$$t = \frac{(\hat{A}_o - A_o)\hat{\sigma}_1\sqrt{n-2}}{\hat{\sigma}\sqrt{\hat{\sigma}_1^2 + \bar{x}^2}} \tag{57}$$

with n-2 degrees of freedom, for:

A_1

$$t = \frac{(\hat{A}_1 - A_1)\hat{\sigma}_1\sqrt{n-2}}{\hat{\sigma}} \tag{58}$$

with n-2 degrees of freedom, where

$$\hat{\sigma} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[Y_i - (\hat{A}_o + \hat{A}_1 X_i)\right]^2} \tag{59}$$

Table 1. Values of independent variables used in the experiment.

Independent Variables	Positive Regression	Horizontal Regression	Negative Regression
X_1	95.0	66.6	27.3
X_2	68.8	19.2	66.4
X_3	86.2	44.4	22.7
X_4	83.5	64.2	14.4
X_5	59.1	70.5	30.0
X_6	11.4	84.0	73.3
X_7	53.0	46.7	99.2
X_8	33.0	26.7	11.0
X_9	20.8	93.7	50.3
X_{10}	4.2	49.5	28.0
X_{11}	83.9	19.5	73.4
X_{12}	93.6	46.6	57.4
X_{13}	31.8	74.3	30.9
X_{14}	55.3	7.1	30.5
X_{15}	43.3	33.3	19.2
X_{16}	46.4	4.2	48.6
X_{17}	86.5	66.4	57.5
X_{18}	54.4	85.5	67.6
X_{19}	63.4	15.2	1.0
X_{20}	72.3	6.3	90.9

and

$$\hat{\sigma}_1 = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n}} \tag{60}$$

and where $\hat{A}_0$ and $\hat{A}_1$ are the estimates of the regression coefficients A_0 and A_1 respectively. The sample size n of each set was 20, but the combined sample size of five sets was 100. Thus, the t-statistics of this experiment had 18 degrees of freedom for each individual sample and 98 degrees of freedom for the combined sample.

The hypotheses tested were for A_0

$$H_0: \hat{A}_0 = A_0$$
$$H_1: \hat{A}_0 \neq A_0$$

and for A_1

$$H_0: \hat{A}_1 = A_1$$
$$H_1: \hat{A}_1 \neq A_1.$$

Thus two-tailed tests were used at the 0.10, 0.05, and 0.01 levels of significance. The resulting statistics are found in the computer output of the appendix (pp. 83-161) and in Tables 2 and 3 as part of the results (pp. 47-52). The Fortran programs for generating the samples, computing the estimates of the regression coefficients, and computing the t-statistics for these estimates are found in the appendix. The programs for individual samples are on pp. 69-76 and the program for the combined samples is on p. 77.

RESULTS

The resulting estimates of A_0 and A_1 based upon samples from the gamma distributions and the corresponding parameters are presented in the first part of Table 2. The computed t-statistics are also shown in this table. Each estimate is based upon a sample size of 20 and there are 90 estimates in all. The differences were tested for significance at three levels--10%, 5%, and 1%. This was done to provide a sensitivity analysis since additional information can be obtained by knowing at what level significant variation occurs. This would not be apparent if a single level of significance was used. The corresponding critical values for the t-statistic with 18 degrees of freedom are 1.734, 2.101, and 2.878. These refer to absolute values since the test being made is a two-tailed test (see previous statement of hypotheses); therefore, the sign of the computed t-statistic in Table 2 should be ignored. Inspection of this table will show that of the 90 estimates only 3 represent significant variation, and all 3 are at the 0.10 level of significance.

The first of these estimates is the case of the gamma distribution with $\alpha = 1$ and $\beta = 1$ or the exponential distribution. The variation is in the estimate made of A_0 for the first set of points with a positively sloped regression function. The population regression function was of the form

$$Y = 1.0 + 2.0(X) \tag{61}$$

where $A_0 = 1.0$ and $A_1 = 2.0$. The estimates based upon the normal

Table 2. Parameters A_o and A_1 with corresponding estimates $\hat{A}_o$ and $\hat{A}_1$ and t-statistics t_{A_o} and t_{A_1} for samples of size 20.

Distributions	A_o	$\hat{A}_o$	t_{A_o}	A_1	$\hat{A}_1$	t_{A_1}
exponential	1.0	0.554	-1.748*	2.0	2.003	0.791
(positive	1.0	1.163	0.301	2.0	1.997	-0.311
regression)	1.0	0.949	-0.105	2.0	2.001	0.203
	1.0	1.315	0.638	2.0	1.993	-0.869
	1.0	1.521	0.795	2.0	1.998	-0.140
exponential	1.0	0.514	-1.424	0.0	0.008	1.416
(horizontal	1.0	1.338	0.685	0.0	-0.003	-0.427
regression)	1.0	0.843	-0.333	0.0	0.003	0.413
	1.0	0.828	-0.364	0.0	0.002	0.333
	1.0	1.307	0.606	0.0	-0.005	-0.548
exponential	1.0	1.383	0.909	-2.0	-2.007	-0.961
(negative	1.0	1.037	0.099	-2.0	-2.002	-0.375
regression)	1.0	0.894	-0.289	-2.0	-2.000+	-0.011
	1.0	0.778	-0.711	-2.0	-1.996	0.665
	1.0	2.119	1.457	-2.0	-2.014	-0.999
chi-square	2.0	1.339	-0.748	2.0	2.012	0.917
(positive	2.0	1.181	-1.068	2.0	2.012	1.054
regression)	2.0	1.742	-0.304	2.0	2.003	0.229
	2.0	1.458	-1.110	2.0	2.006	0.808
	2.0	1.885	-0.170	2.0	2.000+	0.057
chi-square	2.0	1.998	-0.003	0.0	-0.007	-1.115
(horizontal	2.0	2.745	1.216	0.0	-0.016	-1.421
regression)	2.0	1.388	-1.126	0.0	0.014	1.472
	2.0	2.552	0.850	0.0	-0.010	-0.862
	2.0	3.324	1.933*	0.0	-0.014	-1.147
chi-square	2.0	2.234	0.439	-2.0	-2.006	-0.686
(negative	2.0	2.540	0.756	-2.0	-2.007	-0.554
regression	2.0	1.495	-1.102	-2.0	-1.986	1.534
	2.0	2.730	1.101	-2.0	-2.018	-1.475
	2.0	2.308	0.420	-2.0	-1.999	0.015
skewed normal	4.0	2.961	-0.813	2.0	2.018	0.932
(positive	4.0	2.791	-1.113	2.0	2.018	1.098
regression)	4.0	3.341	-0.908	2.0	2.008	0.703
	4.0	3.862	-0.138	2.0	2.000+	0.038
	4.0	2.966	-1.573	2.0	2.010	1.050

* denotes significance at 0.10 level

Table 2. (continued)

Distributions	A_o	$\hat{A}_o$	t_{A_o}	A_1	$\hat{A}_1$	t_{A_1}
skewed normal (horizontal regression)	4.0	3.804	-0.193	0.0	0.011	0.590
	4.0	5.367	1.453	0.0	-0.028	-1.645
	4.0	4.421	0.506	0.0	0.003	-0.239
	4.0	4.826	1.132	0.0	-0.020	-1.497
	4.0	3.902	-0.096	0.0	0.009	0.484
skewed normal (negative regression)	4.0	5.083	1.623	-2.0	-2.021	-1.648
	4.0	2.922	-1.046	-2.0	-2.012	-0.633
	4.0	3.100	-0.988	-2.0	-2.024	-1.433
	4.0	2.379	-1.495	-2.0	-1.999	0.045
	4.0	5.546	1.416	-2.0	-2.016	-0.800
normal variance = 1 (positive regression)	1.0	0.889	-0.218	2.0	1.998	-0.171
	1.0	0.499	-0.905	2.0	1.999	-0.081
	1.0	0.880	-0.214	2.0	2.004	0.528
	1.0	1.139	0.331	2.0	1.999	-0.058
	1.0	1.154	0.525	2.0	1.996	-0.706
normal variance = 1 (horizontal regression)	1.0	1.237	0.484	0.0	-0.002	-0.245
	1.0	0.900	-0.214	0.0	0.007	0.888
	1.0	1.183	0.489	0.0	-0.004	-0.593
	1.0	0.823	-0.441	0.0	0.000+	0.114
	1.0	1.039	0.098	0.0	-0.004	-0.661
normal variance = 1 (negative regression)	1.0	0.977	-0.051	-2.0	-2.004	-0.488
	1.0	0.719	-0.673	-2.0	-1.994	0.643
	1.0	1.573	1.338	-2.0	-2.004	-0.563
	1.0	0.505	-0.861	-2.0	-1.994	0.511
	1.0	1.256	0.794	-2.0	-1.995	0.724
normal variance = 2 (positive regression)	2.0	1.322	-0.582	2.0	2.013	0.710
	2.0	1.728	-0.268	2.0	2.008	0.504
	2.0	3.242	1.048	2.0	1.983	-0.905
	2.0	0.981	-0.930	2.0	2.017	1.032
	2.0	0.655	-1.225	2.0	2.011	0.652
normal variance = 2 (horizontal regression)	2.0	3.336	1.406	0.0	-0.025	-1.442
	2.0	2.295	0.290	0.0	-0.015	-0.819
	2.0	3.373	1.796*	0.0	-0.024	-1.702
	2.0	2.824	1.066	0.0	-0.020	-1.399
	2.0	2.565	0.545	0.0	-0.001	-0.078

* denotes significance at 0.10 level

Table 2. (continued)

Distributions	A_o	$\hat{A}_o$	t_{A_o}	A_1	$\hat{A}_1$	t_{A_1}
normal	2.0	2.071	0.091	-2.0	-2.000+	-0.024
variance = 2	2.0	1.046	-1.222	-2.0	-1.979	1.383
(negative	2.0	1.438	-0.621	-2.0	-2.001	-0.111
regression)	2.0	0.897	-1.452	-2.0	-1.984	1.061
	2.0	1.955	-0.051	-2.0	-2.008	-0.488
normal	4.0	3.026	-0.701	2.0	2.001	0.051
variance = 4	4.0	4.326	0.140	2.0	2.003	0.095
(positive	4.0	1.745	-0.710	2.0	2.037	0.742
regression)	4.0	4.080	0.029	2.0	1.987	-0.291
	4.0	5.334	0.711	2.0	1.969	-1.044
normal	4.0	4.343	0.182	0.0	-0.010	-0.290
variance = 4	4.0	2.807	-0.686	0.0	0.032	1.011
(horizontal	4.0	5.876	1.206	0.0	-0.021	-0.757
regression)	4.0	2.683	-1.007	0.0	-0.011	-0.482
	4.0	5.516	0.771	0.0	-0.038	-1.055
normal	4.0	3.833	-0.108	-2.0	-1.981	0.626
variance = 4	4.0	2.509	-0.825	-2.0	-1.951	1.408
(negative	4.0	3.790	-0.145	-2.0	-2.012	-0.437
regression)	4.0	3.691	-0.170	-2.0	-1.981	0.537
	4.0	5.763	1.151	-2.0	-2.023	-0.791

equations were $\hat{A}_0 = 0.554$ and $\hat{A}_1 = 2.003$. The corresponding t-statistics were $t_{A_0} = -1.748$, which is barely significant at the 10% level, the critical value being 1.734, and $t_{A_1} = 0.791$, which is not significant.

The second significant variation was found in the case of the gamma distribution with $\alpha = 2$ and $\beta = 1$ or the chi-square distribution. It occurred in the fifth set of sample points for the horizontal regression function. The population regression function was of the form

$$Y = 2.0 + 0.0(X) \tag{62}$$

where $A_0 = 2.0$ and $A_1 = 0.0$. The normal equations yielded estimates of $\hat{A}_0 = 3.324$ and $\hat{A}_1 = -0.014$. The corresponding computed t-statistics were $t_{A_0} = 1.933$ and $t_{A_1} = -1.147$. The statistic was again significant only for the estimate of A_0 at the 0.10 level since $t_{A_0} = 1.933$ is greater than $t_{0.10,18} = 1.734$. In this case the difference is less equivocal since the critical region is clearly violated.

The final instance of a significant difference is found in the normal distribution which served as an experimental control. The variation occurred in the case of the normal distribution with variance equal to 2.0; the third set of sample points about the horizontal regression function contains the deviation. The parameters of the regression were

$$Y = 2.0 + 0.0(X) \tag{63}$$

where $A_o = 2.0$ and $A_1 = 0.0$. The estimates of these parameters were $\hat{A}_o = 3.373$ and $\hat{A}_1 = -0.024$. The respective computed t-statistics were $t_{A_o} = 1.796$ and $t_{A_1} = -1.702$. Again only the estimate of A_o can be judged significantly different at the 10% level, since $t_{A_o} = 1.796$ is greater than $t_{0.10,18} = 1.734$. Again the margin for significance is only a slight one.

Table 3 presents estimates of A_o and A_1 and their corresponding t-statistics based upon a combination of the five sets of sample data for each regression function. There are, therefore, eighteen such sets of estimates, each one of which is based upon a sample size of 100. The t-statistics have n-2 or 98 degrees of freedom. The larger sample size makes these tests much more sensitive, and thus would better substantiate the conclusions made concerning non-normal regression. Again three levels of confidence were used in determining significant differences--10%, 5%, and 1%. The critical values are absolute values and are respectively 1.661, 1.982, and 2.625.

An examination of Table 3 will reveal that no significant differences exist at the specified levels of confidence.

It may be noticed in Tables 2 and 3 that in certain cases there appears to be no discrepancy between the estimate of a parameter and the actual value of the parameter, and yet there is a non-zero computed t-statistic. This is explained by the fact that the output format of the computer program was limited to three decimal places and that some of the decimal positions beyond the third one were non-zero. Consequently, a plus sign has been added to those estimates to indicate this fact. An

Table 3. Parameters A_o and A_1 with corresponding estimates of $\hat{A}_o$ and $\hat{A}_1$ and t-statistics t_{A_o} and t_{A_1} for combined samples of size 100.

Distribution	A_o	$\hat{A}_o$	t_{A_o}	A_1	$\hat{A}_1$	t_{A_1}
exponential	1.0	1.101	0.451	2.0	1.999	-0.352
	1.0	0.966	-0.170	0.0	0.001	0.345
	1.0	1.243	1.150	-2.0	-2.004	-1.050
chi-square	2.0	1.521	-1.492	2.0	2.007	1.398
	2.0	2.401	1.527	0.0	-0.007	-1.368
	2.0	2.262	0.944	-2.0	-2.004	-0.739
skewed normal	4.0	3.184	-1.636	2.0	2.011	1.619
	4.0	4.464	1.151	0.0	-0.005	-0.672
	4.0	3.807	-0.409	-2.0	-2.015	-1.643
normal variance = 1	1.0	0.913	-0.410	2.0	2.000+	-0.067
	1.0	1.037	0.194	0.0	0.000+	-0.159
	1.0	1.007	0.034	-2.0	-1.999	0.341
normal variance = 2	2.0	1.586	-0.845	2.0	2.007	0.861
	2.0	2.879	1.196	0.0	-0.017	-1.334
	2.0	1.482	-1.435	-2.0	-1.995	0.741
normal variance = 4	4.0	3.703	-0.287	2.0	2.000+	-0.019
	4.0	4.245	0.321	0.0	-0.010	-0.701
	4.0	3.918	-0.114	-2.0	-1.990	0.732

example of this is $\hat{A}_1 = 2.000$ which has been written as $\hat{A}_1 = 2.000+$. The correctness of this supposition is supported by the very small computed t-statistics which occur in all of the cases. For the example cited the statistic was $t_{A_1} = -0.067$.

DISCUSSION

The three cases of significant variation in the 90 individual samples can be credibly explained. This deviation in the estimates of the regression coefficient A_o occurred for the exponential distribution with a positive regression, the chi-square with a horizontal regression, and the normal distribution with a horizontal regression. The computed t-statistics were respectively 1.748, 1.933, and 1.796 which are significant only at the 90% level of confidence. The critical value $t_{0.10,18} = 1.734$ is very close to each of these statistics. This variation can be explained from the point of view of the significance level $\alpha = 0.10$. This level stipulates that 10% of the time the null hypothesis H_o: $\hat{A}_o = A_o$ will be rejected when it is actually true or that the probability of a Type I error is 0.10. This view is substantiated by the fact that of the 90 individual samples only the three mentioned above resulted in rejection of the null hypothesis; whereas, with $\alpha = 0.10$, one could reasonably expect to reject the null hypothesis based upon 9 (0.10 X 90) of these samples before concluding that a cause other than the possibility of a Type I error was at work.

Furthermore, the combination of these individual sample sets in groups of five so as to make an overall sample size of 100 yielded estimates of the regression coefficients which did not vary significantly. It should be noted that these larger sample sets included the individual sets which had resulted in the significant variation of the estimates from the true parameters.

Since the greater sample size provides a much more sensitive test, these latter results are more conclusive than the statistics based upon the smaller sample size. This condition also attests to the possibility of the above-mentioned Type I error, since in the case of the combined samples one would expect to reject a true null hypothesis on the basis of two such samples for $\alpha = 0.10$ ($0.10 \times 18 = 1.8 \approx 2$).

Since the exponential distribution is the most skewed of the non-normal distributions, it would be expected to cause significant variation in estimating parameters based on a normal distribution if any variation were encountered at all. Therefore, if variation can be reasonably explained in the case of the exponential distribution, it can certainly be explained in the case of the less skewed distributions.

This theory meets with one objection, however; although no formal analysis was made, the deviation of estimated values from actual values and the resulting t-statistics seem to be independent of the distribution used or the type of regression function assumed--positively, zero, or negatively sloped. The only exception to this is that in both Tables 2 and 3 the less skewed chi-square and skewed normal distribution have generally resulted in larger t-statistics than the more skewed exponential distribution. This of course is due to larger discrepancies between the estimated parameters and the true parameters. On the surface this appears to be a contradiction; however, one should recall Fig. 18 and the discussion relating to it. These two distributions had distribution functions which could not be solved

explicitly for the dependent variable y. An iterative technique was used and although an accuracy of 0.00001 was provided in the probability of y, i.e., F(y), such a discrepancy may result in a greater error for large values of y. This may well have been a factor in the association of larger t-statistics with these distributions.

One other trend which should be noted in Tables 2 and 3 is that the estimates of the regression slope A_1 were usually closer to the true parameters than were the $\hat{A}_o$'s to the corresponding A_o's. In turn the statistic t_{A_1} was generally smaller than the corresponding t_{A_o}. This should be expected, from the manner in which the sample was taken. The distributions had constant slope throughout the range of X, and the sample would be expected to vary as the slope varied with X. Nor should the statistic t_{A_o} be as sensitive to variation as the statistic t_{A_1} is, since $\hat{A}_o$ is a point estimate and $\hat{A}_1$ is an estimate of the slope. An example of this is $\hat{A}_o = 3.184$ and $A_o = 4.000$ with $|t_{A_o}| = |1.636|$ and $\hat{A}_1 = 2.011$ and $A_1 = 2.000$ with $t_{A_1} = 1.619$. A small error in estimating A_1 results in the same size t-statistic as does a much larger error made in estimating A_o.

Another noteworthy aspect of the results found in the appendix (pp. 83-161) is that for the various gamma regressions with negative slopes a negative dependent variable Y was generated. Since the range of the gamma distribution is from 0 to ∞, this appears to be an inconsistency. This occurred because the values of the gamma distribution were translated so that negative values would appear.

CONCLUSIONS

The principle conclusion of this study is that the regression function of Model I which is computed from data assumed to be normally distributed is a satisfactory representation of the true regression function when the data is in fact from a non-normal distribution. The hypotheses

$$H_0: \; A_0(\text{non-normal}) = A_0(\text{normal})$$

and

$$H_0: \; A_1(\text{non-normal}) = A_1(\text{normal})$$

cannot be rejected on the evidence this study presents. This conclusion applies to the case of simple linear regression with a dependent variable distributed according to the gamma distribution. This conclusion is made at the 0.10, 0.05, and 0.01 levels of significance.

Caution should be used in extending this conclusion to other regression functions, other non-normal distributions, or to Model II of regression analysis. It should be remembered that Gumbel's (9) study, which was cited in the literature survey, indicated a striking difference between the regression equations of Model II for conditions of normally distributed and non-normally distributed dependent variables (p. 19). Consequently, it is strongly recommended that similar studies be made for the cases of non-linear and multiple regression functions. In addition Model II should be considered for these cases and for the simple linear case. Gumbel's concluding remarks bear out this approach.

The fact that many of the properties of the bivariate

normal distributions do not hold here may serve as a warning against the indiscriminate use of normal correlation and regression analysis; prior investigation of the nature of the bivariate distributions is necessary.[5]

[5]E. J. Gumbel, "Bivariate Exponential Distributions", _Journal of the American Statistical Association_, 55(1960), p. 707.

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APPENDIX

Integration by Parts of Distribution Functions

$$f(y|x) = \frac{(y - A_1x)^{\alpha - 1}}{\beta^{\alpha}\Gamma(\alpha)} e^{-(y - A_1x)/\beta}$$

$$F(y|x) = \int_{A_1x}^{y} \frac{(y - A_1x)^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e^{-(y - A_1x)/\beta} dy$$

$$F(y|x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} \int_{A_1x}^{y} (y - A_1x)^{\alpha-1} e^{-(y - A_1x)/\beta} dy \qquad (1)$$

The algorithm of integration by parts states:

$$u.dv = u.v - v.du$$

therefore for $\alpha = 1, \beta = 1$ (exponential distribution function)

$$F(y|x) = \int_{A_1x}^{y} e^{-(y - A_1x)} dy = 1 - e^{-(y - A_1x)}$$

from (1)

$$u = (y - A_1x)^{\alpha-1} \qquad du = (\alpha - 1)(y - A_1x)^{\alpha-2} dy$$

$$v = -\beta e^{-(y - A_1x)/\beta} \qquad dv = e^{-(y - A_1x)/\beta} dy$$

therefore

$$F(y|x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} \Big[(-\beta)(y - A_1x)^{\alpha-1} e^{-(y - A_1X)/\beta} +$$

$$(\alpha - 1)\beta \int_{A_1x}^{y} (y - A_1x)^{-2} e^{-(y - A_1x)/\beta} dy\Big]$$

$$F(y|x) = \frac{-(y - A_1x)\, e^{-(y - A_1x)/\beta}}{\beta^{\alpha-1}\Gamma(\alpha)} + \frac{\alpha - 1}{\beta^{\alpha-1}\Gamma(\alpha)} \int_{A_1x}^{y} (y - A_1x)^{\alpha-2} \cdot$$

$$e^{-(y - A_1x)/\beta} dy \qquad (2)$$

therefore for $\alpha = 2, \beta = 1$ (chi-square distribution)

$$F(y|x) = 1 - (y - A_1x)\, e^{-(y - A_1x)} - e^{-(y - A_1x)}$$

from (2)

$$u = (y - A_1x)^{\alpha-2} \qquad du = (\alpha-2)\ (y - A_1x)^{\alpha-3}\ dy$$

$$v = -\beta e^{-(y - A_1x)/\beta} \qquad dv = e^{-(y - A_1x)/\beta}\ dy$$

therefore

$$F(y|x) = \frac{-(y - A_1x)^{\alpha-1}\ e^{-(y - A_1x)/\beta}}{\beta^{\alpha-1}\Gamma(\alpha)} + \frac{\alpha - 1}{\beta^{\alpha-1}\Gamma(\alpha)}\Big[(-\beta)\ (y - A_1x)$$

$$\exp\ (\alpha - 2)\ + \beta(\alpha - 2) \int_{A_1x}^{y} (y - A_1x)^{\alpha-3}\ e^{-(y - A_1x)/\beta}$$

$$dy\Big]$$

$$F(y|x) = \frac{-(y - A_1x)^{\alpha-1}\ e^{-(y - A_1x)/\beta}}{\beta^{\alpha-1}\Gamma(\alpha)} - \frac{(\alpha - 1)(y - A_1x)^{\alpha-2}\ e}{\beta^{\alpha-2}\Gamma(\alpha)}$$

$$\exp\ -(y - A_1x)/\beta + \frac{(\alpha-1)\ (\alpha - 2)}{\beta^{\alpha-2}\Gamma(\alpha)} \int_{A_1x}^{y} (y - A_1x)^{\alpha-3}\ .$$

$$e^{-(y - A_1x)/\beta}\ dy \tag{3}$$

from (3)

$$u = (y - A_1x)^{\alpha-3} \qquad du = (\alpha - 3)\ (y - A_1x)^{\alpha-4}\ dy$$

$$v = -\beta e^{-(y - A_1x)/\beta} \qquad dv = e^{-(y - A_1x)/\beta} dy$$

therefore

$$F(y|x) = \frac{-(y - A_1x)^{\alpha-1}\ e^{-(y - A_1x)/\beta}}{\beta^{\alpha-1}\Gamma(\alpha)} - \frac{(\alpha - 1)(y - A_1x)^{\alpha-2}\ e}{\beta^{\alpha-2}\Gamma(\alpha)}$$

$$\exp(-(y - A_1x)/\beta) - \frac{(\alpha - 1)(\alpha - 2)(y - A_1x)^{\alpha-3}\ e}{\beta^{\alpha-3}\Gamma(\alpha)}$$

$$\exp(-(y - A_1x)/\beta) + \frac{(\alpha - 1)(\alpha - 2)(\alpha - 3)}{\beta^{\alpha-3}\Gamma(\alpha)} \int_{A_1x}^{y} (y - A_1x)$$

$$\exp(\alpha - 4)\ e^{-(y-A_1x)/\beta} dy$$

for $\alpha = 4, \beta = 1$ (negatively skewed normal distribution)

$$F(y|x) = 1 - \frac{(y - A_1x)^3}{6} e^{-(y - A_1x)} - \frac{(y - A_1x)^2}{2} e^{-(y - A_1x)}$$

$$- (y - A_1x) e^{-(y - A_1x)} - e^{-(y - A_1x)}$$

** KENDALL, BABINGTON-SMITH TESTS FOR RANDOMNESS

```
      DIMENSION F(20),FF(20,20)
      READ 20,A1,SEQ,K
   20 FORMAT(F5.0,1XF5.0,1XI3)
      T=K
      DO 5 I=1,K
      F(I)=0.
    5 CONTINUE
      DO 17 J=1,K
      DO 50 I=1,K
      FF(J,I)=0.
   50 CONTINUE
   17 CONTINUE
      C=0.
  100 A2=RNDM(A1)
      IF(SENSE SWITCH1)40,1
   40 PRINT41,A2
   41 FORMAT(F12.9)
    1 C=C+1.
      I=0
   63 I=I+1
      D=I
      IF(A2-(1./T)*D)64,64,63
   64 F(I)=F(I)+1.0
      IF(C-1.0)7,7,21
   21 FF(J,I)=FF(J,I)+1.
    7 J=I
      IF(C-SEQ)100,16,15
   16 X1=0.
      DO 6 I=1,K
      X=(F(I)-C/T)*(F(I)-C/T)
      X1=X1+X
    6 CONTINUE
      X12=T/C*X1
      X2=0.
      DO 18 J=1,K
      DO 70 I=1,K
      Y=(FF(J,I)-C/(T*T))*(FF(J,I)-C/(T*T))
      X2=X2+Y
   70 CONTINUE
   18 CONTINUE
      X22=T*T/C*X2
      PUNCH 19,X12,X22
   19 FORMAT(14HCHI-SQUARE 1 =F12.3,3X14HCHI-SQUARE 2 =F12.3)
      DO 31 I=1,K
      PUNCH 30,I,F(I)
   30 FORMAT(2HF I2,1X2H= F6.1)
   31 CONTINUE
      DO 33 J=1,K
      DO 60 I=1,K
      PUNCH 32,J,I,FF(J,I)
```

```
32 FORMAT(2HF(I2,1H,I2,2H)=F6.1)
60 CONTINUE
33 CONTINUE
   PUNCH42,C
42 FORMAT(F6.1)
15 STOP
   END
```

FORTRAN TEST PROGRAM FOR RANDOM NUMBER GENERATOR
SUBROUTINE-STANDARD NORMAL VARIATES

```
      DIMENSION MFREQ(14),CHI(14),NFREQ(14),CHISQ(14),CHISS(14)
      READ 102,A1,NMAX
  102 FORMAT (F5.0,1XI5)
      T=NMAX
      SUMCH=0.
      SUMMF=0.
      SUMNF=0.
      EP1=-3.99
      EP2=-3.50
      NFREQ(1)=0.0013*T+0.5
      NFREQ(2)=0.0049*T+0.5
      NFREQ(3)=0.0166*T+0.5
      NFREQ(4)=0.0440*T+0.5
      NFREQ(5)=0.0919*T+0.5
      NFREQ(6)=0.1498*T+0.5
      NFREQ(7)=0.1915*T+0.5
      NFREQ(8)=NFREQ(7)
      NFREQ(9)=NFREQ(6)
      NFREQ(10)=NFREQ(5)
      NFREQ(11)=NFREQ(4)
      NFREQ(12)=NFREQ(3)
      NFREQ(13)=NFREQ(2)
      NFREQ(14)=NFREQ(1)
    1 DO 2 I=1,14
    2 MFREQ(I)=0
      NCTR=0
   20 X=0.
      DO 40 I=1,12
      X=X+RNDM(A1)
   40 CONTINUE
      X=X-6.
      IF(SENSE SWITCH 1)50,52
   50 PRINT 51,X
   51 FORMAT(F8.3)
   52 J=0
      CLASS=-3.50
   12 J=J+1
      CLASS=CLASS+0.5
      IF(CLASS-X)11,5,5
   11 IF(J-14)12,5,13
    5 MFREQ(J)=MFREQ(J)+1
      NCTR=NCTR+1
      IF(NCTR-NMAX)20,8,8
    8 DO 10 I=1,14
      CHI(I)=MFREQ(I)-NFREQ(I)
      F=NFREQ(I)
      FF=MFREQ(I)
      CHISS(I)=CHI(I)**2
      IF(F-0.)31,30,31
```

```
 31 CHISQ(I)=CHISS(I)/F
    SUMCH=SUMCH+CHISQ(I)
 30 SUMMF=SUMMF+FF
    SUMNF=SUMNF+F
    EP1=EP1+0.5
    EP2=EP2+0.5
 10 PUNCH 100,EP1,EP2,MFREQ(I),NFREQ(I),CHISS(I),CHISQ(I)
100 FORMAT(F5.2,2H-(F5.2,1H)5XI6,5XI6,5XF7.1,5XF8.3)
    PUNCH 101,SUMMF,SUMNF,SUMCH
101 FORMAT(F25.0,F10.0,F25.3)
 13 STOP
    END
```

PROGRAM FOR ESTIMATING A0 AND A1 WITH T-TESTS
FOR EXPONENTIAL DISTRIBUTION

```
      READ 20,M
   20 FORMAT(I4)
      READ 1,AZERO,AONE,N,A1
    1 FORMAT(F6.1,1XF6.1,1XI2,1XF5.0)
      DIMENSION X(100),Y(100)
      DO 2 I=1,N
      READ 3,X(I)
    3 FORMAT(F6.1)
    2 CONTINUE
      DO 21 J=1,M
      DO 4 I=1,N
      Z=RNDM(A1)
      IF(SENSE SWITCH 1)50,52
   50 TYPE 51,Z
   51 FORMAT(F10.8)
   52 Y(I)=AONE*X(I)-LOGF(1.-Z)
    4 CONTINUE
      T=N
      SUMX=0.
      SUMY=0.
      SUMXY=0.
      SUMX2=0.
      DO 5 I=1,N
      SUMX=SUMX+X(I)
      SUMY=SUMY+Y(I)
      XY=X(I)*Y(I)
      SUMXY=SUMXY+XY
      X2=X(I)**2
      SUMX2=SUMX2+X2
    5 CONTINUE
      SUM2X=SUMX**2
      XBAR=SUMX/T
      A0EST=(SUMX2*SUMY-SUMX*SUMXY)/(T*SUMX2-SUM2X)
      A1EST=(T*SUMXY-SUMX*SUMY)/(T*SUMX2-SUM2X)
      SSX=0.
      DO 6 I=1,N
      SSX2=(X(I)-XBAR)**2
      SSX=SSX+SSX2
    6 CONTINUE
      SIGX=SQRTF(SSX/T)
      SIGX2=SIGX**2
      SSXY=0.
      DO 7 I=1,N
      SSXY2=(Y(I)-A0EST-A1EST*X(I))**2
      SSXY=SSXY+SSXY2
    7 CONTINUE
      SIGXY=SQRTF(SSXY/T)
      TA1=((A1EST-AONE)*SIGX*SQRTF(T-2.))/SIGXY
      TA0=((A0EST-AZERO)*SIGX*SQRTF(T-2.))/(SIGXY*SQRTF(SIGX2+XBAR**2))
```

```
      PUNCH 8,AZERO,A0EST,AONE,A1EST
    8 FORMAT(3HA0=F6.1,1X7HEST A0=F8.3,1X3HA1=F6.1,1X7HEST A1=F8.3)
      DO 9 I=1,N
      PUNCH10,I,X(I),I,Y(I)
   10 FORMAT(2HX(I2,2H)=F6.1,3X2HY(I2,2H)=F12.3)
    9 CONTINUE
      PUNCH 11,TA0,TA1
   11 FORMAT(6HT(A0)=F10.3,1X6HT(A1)=F10.3)
   21 CONTINUE
      END
```

PROGRAM FOR ESTIMATING A0 AND A1 WITH T-TESTS
FOR CHI-SQUARE DISTRIBUTION

```
      READ 20,M
   20 FORMAT(I4)
      READ 1,AZERO,AONE,N,A1
    1 FORMAT(F6.1,1XF6.1,1XI2,1XF5.0)
      DIMENSION X(100),Y(100)
      DO 2 I=1,N
      READ 3,X(I)
    3 FORMAT(F6.1)
    2 CONTINUE
      DO 21 J=1,M
      DO 4 I=1,N
      Z=RNDM(A1)
      IF(SENSE SWITCH 1)50,52
   50 TYPE 51,Z
   51 FORMAT(F10.8)
   52 ALOW=AONE*X(I)
      AHIGH=10.0+AONE*X(I)
  210 Y(I)=(ALOW+AHIGH)/2.
      ZCAL=(AONE*X(I)-Y(I))*EXPF(AONE*X(I)-Y(I))-EXPF(AONE*X(I)-Y(I))+1
      ER=ZCAL-Z
      IF(ER)201,4,200
  201 ER=ER*(-1.)
      IF(ER-0.00001)4,4,202
  202 ALOW=Y(I)
      GO TO 210
  200 IF(ER-0.00001)4,4,203
  203 AHIGH=Y(I)
      GO TO 210
    4 CONTINUE
      T=N
      SUMX=0.
      SUMY=0.
      SUMXY=0.
      SUMX2=0.
      DO 5 I=1,N
      SUMX=SUMX+X(I)
      SUMY=SUMY+Y(I)
      XY=X(I)*Y(I)
      SUMXY=SUMXY+XY
      X2=X(I)**2
      SUMX2=SUMX2+X2
    5 CONTINUE
      SUM2X=SUMX**2
      XBAR=SUMX/T
      A0EST=(SUMX2*SUMY-SUMX*SUMXY)/(T*SUMX2-SUM2X)
      A1EST=(T*SUMXY-SUMX*SUMY)/(T*SUMX2-SUM2X)
      SSX=0.
      DO 6 I=1,N
      SSX2=(X(I)-XBAR)**2
```

```
      SSX=SSX+SSX2
    6 CONTINUE
      SIGX=SQRTF(SSX/T)
      SIGX2=SIGX**2
      SSXY=0.
      DO 7 I=1,N
      SSXY2=(Y(I)-A0EST-A1EST*X(I))**2
      SSXY=SSXY+SSXY2
    7 CONTINUE
      SIGXY=SQRTF(SSXY/T)
      TA1=((A1EST-AONE)*SIGX*SQRTF(T-2.))/SIGXY
      TA0=((A0EST-AZERO)*SIGX*SQRTF(T-2.))/(SIGXY*SQRTF(SIGX2+XBAR**2))
      PUNCH 8,AZERO,A0EST,AONE,A1EST
    8 FORMAT(3HA0=F6.1,1X7HEST A0=F8.3,1X3HA1=F6.1,1X7HEST A1=F8.3)
      DO 9 I=1,N
      PUNCH10,I,X(I),I,Y(I)
   10 FORMAT(2HX(I2,2H)=F6.1,3X2HY(I2,2H)=F12.3)
    9 CONTINUE
      PUNCH 11,TA0,TA1
   11 FORMAT(6HT(A0)=F10.3,1X6HT(A1)=F10.3)
   21 CONTINUE
      END
```

PROGRAM FOR ESTIMATING A0 AND A1 WITH T-TESTS
FOR SKEWED NORMAL DISTRIBUTION

```
      READ 20,M
   20 FORMAT(I4)
      READ 1,AZERO,AONE,N,A1
    1 FORMAT(F6.1,1XF6.1,1XI2,1XF5.0)
      DIMENSION X(100),Y(100)
      DO 2 I=1,N
      READ 3,X(I)
    3 FORMAT(F6.1)
    2 CONTINUE
      DO 21 J=1,M
      DO 4 I=1,N
      Z=RNDM(A1)
      IF(SENSE SWITCH 1)50,52
   50 TYPE 51,Z
   51 FORMAT(F10.8)
   52 ALOW=AONE*X(I)
      AHIGH=20.0+AONE*X(I)
  210 Y(I)=(ALOW+AHIGH)/2.
      ZCAL=(-1.)*(Y(I)-AONE*X(I))**3*EXPF(AONE*X(I)-Y(I))/6.0-(Y(I)-AONE
     1*X(I))**2*EXPF(AONE*X(I)-Y(I))/2.0+(AONE*X(I)-Y(I))*EXPF(AONE*X(I)
     1-Y(I))-EXPF(AONE*X(I)-Y(I))+1.0
      ER=ZCAL-Z
      IF(ER)201,4,200
  201 ER=ER*(-1.)
      IF(ER-0.00001)4,4,202
  202 ALOW=Y(I)
      GO TO 210
  200 IF(ER-0.00001)4,4,203
  203 AHIGH=Y(I)
      GO TO 210
    4 CONTINUE
      T=N
      SUMX=0.
      SUMY=0.
      SUMXY=0.
      SUMX2=0.
      DO 5 I=1,N
      SUMX=SUMX+X(I)
      SUMY=SUMY+Y(I)
      XY=X(I)*Y(I)
      SUMXY=SUMXY+XY
      X2=X(I)**2
      SUMX2=SUMX2+X2
    5 CONTINUE
      SUM2X=SUMX**2
      XBAR=SUMX/T
      A0EST=(SUMX2*SUMY-SUMX*SUMXY)/(T*SUMX2-SUM2X)
      A1EST=(T*SUMXY-SUMX*SUMY)/(T*SUMX2-SUM2X)
      SSX=0.
```

```
      DO 6 I=1,N
      SSX2=(X(I)-XBAR)**2
      SSX=SSX+SSX2
    6 CONTINUE
      SIGX=SQRTF(SSX/T)
      SIGX2=SIGX**2
      SSXY=0.
      DO 7 I=1,N
      SSXY2=(Y(I)-A0EST-A1EST*X(I))**2
      SSXY=SSXY+SSXY2
    7 CONTINUE
      SIGXY=SQRTF(SSXY/T)
      TA1=((A1EST-AONE)*SIGX*SQRTF(T-2.))/SIGXY
      TA0=((A0EST-AZERO)*SIGX*SQRTF(T-2.))/(SIGXY*SQRTF(SIGX2+XBAR**2))
      PUNCH 8,AZERO,A0EST,AONE,A1EST
    8 FORMAT(3HA0=F6.1,1X7HEST A0=F8.3,1X3HA1=F6.1,1X7HEST A1=F8.3)
      DO 9 I=1,N
      PUNCH10,I,X(I),I,Y(I)
   10 FORMAT(2HX(I2,2H)=F6.1,3X2HY(I2,2H)=F12.3)
    9 CONTINUE
      PUNCH 11,TA0,TA1
   11 FORMAT(6HT(A0)=F10.3,1X6HT(A1)=F10.3)
   21 CONTINUE
      END
```

PROGRAM FOR ESTIMATING A0 AND A1 WITH T-TESTS
FOR ALL NORMAL DISTRIBUTIONS

```
      READ 53,SIGMA
   53 FORMAT(F6.1)
      READ 20,M
   20 FORMAT(I4)
      DIMENSION X(100),Y(100)
      READ 1,AZERO,AONE,N,A1
    1 FORMAT(F6.1,1XF6.1,1XI2,1XF5.0)
      DO 2 I=1,N
      READ 3,X(I)
    3 FORMAT(F6.1)
    2 CONTINUE
      DO 21 J=1,M
      TMU=AZERO
      DO 4 I=1,N
      Z=0.
      DO 54 K=1,12
      Z=Z+RNDM(A1)
   54 CONTINUE
      Z=Z-6.
      IF(SENSE SWITCH 1)50,52
   50 TYPE 51,Z
   51 FORMAT(F8.3)
   52 Y(I)=Z*SIGMA+TMU+AONE*X(I)
    4 CONTINUE
      T=N
      SUMX=0.
      SUMY=0.
      SUMXY=0.
      SUMX2=0.
      DO 5 I=1,N
      SUMX=SUMX+X(I)
      SUMY=SUMY+Y(I)
      XY=X(I)*Y(I)
      SUMXY=SUMXY+XY
      X2=X(I)**2
      SUMX2=SUMX2+X2
    5 CONTINUE
      SUM2X=SUMX**2
      XBAR=SUMX/T
      A0EST=(SUMX2*SUMY-SUMX*SUMXY)/(T*SUMX2-SUM2X)
      A1EST=(T*SUMXY-SUMX*SUMY)/(T*SUMX2-SUM2X)
      SSX=0.
      DO 6 I=1,N
      SSX2=(X(I)-XBAR)**2
      SSX=SSX+SSX2
    6 CONTINUE
      SIGX=SQRTF(SSX/T)
      SIGX2=SIGX**2
      SSXY=0.
```

```
      DO 7 I=1,N
      SSXY2=(Y(I)-A0EST-A1EST*X(I))**2
      SSXY=SSXY+SSXY2
    7 CONTINUE
      SIGXY=SQRTF(SSXY/T)
      TA1=((A1EST-AONE)*SIGX*SQRTF(T-2.))/SIGXY
      TA0=((A0EST-AZERO)*SIGX*SQRTF(T-2.))/(SIGXY*SQRTF(SIGX2+XBAR**2))
      PUNCH 8,AZERO,A0EST,AONE,A1EST
    8 FORMAT(3HA0=F6.1,1X7HEST A0=F8.3,1X3HA1=F6.1,1X7HEST A1=F8.3)
      DO 9 I=1,N
      PUNCH10,I,X(I),I,Y(I)
   10 FORMAT(2HX(I2,2H)=F6.1,3X2HY(I2,2H)=F12.3)
    9 CONTINUE
      PUNCH 11,TA0,TA1
   11 FORMAT(6HT(A0)=F10.3,1X6HT(A1)=F10.3)
   21 CONTINUE
      END
```

PROGRAM FOR COMBINED SAMPLE ESTIMATES OF A0 AND A1
WITH T-TESTS FOR ALL DISTRIBUTIONS

```
      DIMENSION X(100),Y(100)
      READ 1,AZERO,AONE,N
    1 FORMAT(F6.1,1XF6.1,1XI3)
      DO 2 I=1,N
      READ 3,X(I),Y(I)
    3 FORMAT(6XF6.1,12XF9.3)
    2 CONTINUE
      T=N
      SUMX=0.
      SUMY=0.
      SUMXY=0.
      SUMX2=0.
      DO 5 I=1,N
      SUMX=SUMX+X(I)
      SUMY=SUMY+Y(I)
      XY=X(I)*Y(I)
      SUMXY=SUMXY+XY
      X2=X(I)**2
      SUMX2=SUMX2+X2
    5 CONTINUE
      SUM2X=SUMX**2
      XBAR=SUMX/T
      A0EST=(SUMX2*SUMY-SUMX*SUMXY)/(T*SUMX2-SUM2X)
      A1EST=(T*SUMXY-SUMX*SUMY)/(T*SUMX2-SUM2X)
      SSX=0.
      DO 6 I=1,N
      SSX2=(X(I)-XBAR)**2
      SSX=SSX+SSX2
    6 CONTINUE
      SIGX=SQRTF(SSX/T)
      SIGX2=SIGX**2
      SSXY=0.
      DO 7 I=1,N
      SSXY2=(Y(I)-A0EST-A1EST*X(I))**2
      SSXY=SSXY+SSXY2
    7 CONTINUE
      SIGXY=SQRTF(SSXY/T)
      TA1=((A1EST-AONE)*SIGX*SQRTF(T-2.))/SIGXY
      TA0=((A0EST-AZERO)*SIGX*SQRTF(T-2.))/(SIGXY*SQRTF(SIGX2+XBAR**2))
      PUNCH 8,AZERO,A0EST,AONE,A1EST
    8 FORMAT(3HA0=F6.1,1X7HEST A0=F8.3,1X3HA1=F6.1,1X7HEST A1=F8.3)
      DO 9 I=1,N
      PUNCH10,I,X(I),I,Y(I)
   10 FORMAT(2HX(I3,2H)=F6.1,3X2HY(I3,2H)=F12.3)
    9 CONTINUE
      PUNCH 11,TA0,TA1
   11 FORMAT(6HT(A0)=F10.3,1X6HT(A1)=F10.3)
      END
```

RESULTS OF THE KENDALL, BABINGTON-SMITH TESTS FOR RANDOMNESS

```
CHI-SQUARE 1 =        3.960   CHI-SQUARE 2 =      103.800
F  1 =   46.0
F  2 =   61.0
F  3 =   51.0
F  4 =   48.0
F  5 =   45.0
F  6 =   51.0
F  7 =   51.0
F  8 =   45.0
F  9 =   52.0
F 10 =   50.0
F( 1, 1)=   4.0
F( 1, 2)=   7.0
F( 1, 3)=   7.0
F( 1, 4)=   5.0
F( 1, 5)=   5.0
F( 1, 6)=   6.0
F( 1, 7)=   3.0
F( 1, 8)=   2.0
F( 1, 9)=   3.0
F( 1,10)=   4.0
F( 2, 1)=   8.0
F( 2, 2)=   6.0
F( 2, 3)=   5.0
F( 2, 4)=   8.0
F( 2, 5)=   4.0
F( 2, 6)=   5.0
F( 2, 7)=   9.0
F( 2, 8)=   6.0
F( 2, 9)=   4.0
F( 2,10)=   6.0
F( 3, 1)=   6.0
F( 3, 2)=   6.0
F( 3, 3)=   5.0
F( 3, 4)=   4.0
F( 3, 5)=   5.0
F( 3, 6)=   6.0
F( 3, 7)=   9.0
F( 3, 8)=   3.0
F( 3, 9)=   6.0
F( 3,10)=   1.0
F( 4, 1)=   5.0
F( 4, 2)=   6.0
F( 4, 3)=   7.0
F( 4, 4)=   7.0
F( 4, 5)=   0.0
F( 4, 6)=   7.0
F( 4, 7)=   3.0
F( 4, 8)=   2.0
F( 4, 9)=   4.0
```

```
F( 4,10)=   7.0
F( 5, 1)=   2.0
F( 5, 2)=   3.0
F( 5, 3)=   5.0
F( 5, 4)=   4.0
F( 5, 5)=   3.0
F( 5, 6)=   6.0
F( 5, 7)=   3.0
F( 5, 8)=   9.0
F( 5, 9)=   3.0
F( 5,10)=   7.0
F( 6, 1)=   6.0
F( 6, 2)=   5.0
F( 6, 3)=   5.0
F( 6, 4)=   7.0
F( 6, 5)=   6.0
F( 6, 6)=   5.0
F( 6, 7)=   5.0
F( 6, 8)=   2.0
F( 6, 9)=   3.0
F( 6,10)=   7.0
F( 7, 1)=   1.0
F( 7, 2)=   5.0
F( 7, 3)=   4.0
F( 7, 4)=   3.0
F( 7, 5)=   4.0
F( 7, 6)=   2.0
F( 7, 7)=   6.0
F( 7, 8)=   4.0
F( 7, 9)=  15.0
F( 7,10)=   7.0
F( 8, 1)=   5.0
F( 8, 2)=  10.0
F( 8, 3)=   2.0
F( 8, 4)=   2.0
F( 8, 5)=   7.0
F( 8, 6)=   3.0
F( 8, 7)=   4.0
F( 8, 8)=   6.0
F( 8, 9)=   4.0
F( 8,10)=   2.0
F( 9, 1)=   4.0
F( 9, 2)=  10.0
F( 9, 3)=   4.0
F( 9, 4)=   3.0
F( 9, 5)=   6.0
F( 9, 6)=   5.0
F( 9, 7)=   5.0
F( 9, 8)=   8.0
F( 9, 9)=   5.0
F( 9,10)=   1.0
F(10, 1)=   5.0
F(10, 2)=   2.0
```

```
F(10, 3)=    7.0
F(10, 4)=    5.0
F(10, 5)=    5.0
F(10, 6)=    6.0
F(10, 7)=    4.0
F(10, 8)=    3.0
F(10, 9)=    5.0
F(10,10)=    8.0
 500.0
```

RESULTS FOR THE CHI-SQUARE STANDARD NORMAL DISTRIBUTION
GOODNESS OF FIT TEST

-3.49-(-3.00)	1	1	0.0	0.000
-2.99-(-2.50)	5	5	0.0	0.000
-2.49-(-2.00)	8	17	81.0	4.764
-1.99-(-1.50)	35	44	81.0	1.840
-1.49-(-1.00)	97	92	25.0	.271
-.99-(-.50)	149	150	1.0	.006
-.49-(0.00)	186	192	36.0	.187
.01-(.50)	192	192	0.0	0.000
.51-(1.00)	147	150	9.0	.060
1.01-(1.50)	103	92	121.0	1.315
1.51-(2.00)	50	44	36.0	.818
2.01-(2.50)	18	17	1.0	.058
2.51-(3.00)	7	5	4.0	.800
3.01-(3.50)	2	1	1.0	1.000
	1000.	1002.		11.123

SAMPLE OUTPUT OF RANDOM UNIFORM VARIABLES DISTRIBUTED BETWEEN 0 AND 1

.27340083
.21990168
.38352928
.45166376
.68966635
.10470304
.75429976
.07061040
.45165987
.19340840
.32974224
.83285304
.17858139
.75161776
.05065672
.55279168
.36963091
.88493112
.97784320
.20482632
.40400843
.53894848
.29210168
.92335696
.54091395
.49926984
.29423216
.00278360
.91954747
.39149520
.40503464
.89750624
.55910899
.68122456
.16530912
.22192488
.35879351
.67405792
.23585560
.75043952
.09781603
.51559528
.39747408
.41745016
.43536155
.19143664
.55096456
.31735680
.91063507
.52718200

```
INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR EXPONENTIAL WITH POSITIVE REGRESSION


A0=   1.0 EST A0=     .554 A1=   2.0 EST A1=   2.003
X( 1)=  95.0   Y( 1)=      191.171
X( 2)=  68.8   Y( 2)=      138.350
X( 3)=  86.2   Y( 3)=      172.738
X( 4)=  83.5   Y( 4)=      167.738
X( 5)=  59.1   Y( 5)=      118.981
X( 6)=  11.4   Y( 6)=       22.986
X( 7)=  53.0   Y( 7)=      107.200
X( 8)=  33.0   Y( 8)=       66.590
X( 9)=  20.8   Y( 9)=       41.626
X(10)=   4.2   Y(10)=        8.637
X(11)=  83.9   Y(11)=      168.596
X(12)=  93.6   Y(12)=      188.331
X(13)=  31.8   Y(13)=       64.879
X(14)=  55.3   Y(14)=      110.820
X(15)=  43.3   Y(15)=       88.304
X(16)=  46.4   Y(16)=       94.194
X(17)=  86.5   Y(17)=      173.092
X(18)=  54.4   Y(18)=      109.522
X(19)=  63.4   Y(19)=      127.734
X(20)=  72.3   Y(20)=      145.051
T(A0)=      -1.748 T(A1)=         .791

A0=   1.0 EST A0=    1.163 A1=   2.0 EST A1=   1.997
X( 1)=  95.0   Y( 1)=      190.622
X( 2)=  68.8   Y( 2)=      138.088
X( 3)=  86.2   Y( 3)=      172.547
X( 4)=  83.5   Y( 4)=      167.455
X( 5)=  59.1   Y( 5)=      118.270
X( 6)=  11.4   Y( 6)=       24.924
X( 7)=  53.0   Y( 7)=      107.378
X( 8)=  33.0   Y( 8)=       67.487
X( 9)=  20.8   Y( 9)=       41.601
X(10)=   4.2   Y(10)=       10.717
X(11)=  83.9   Y(11)=      170.117
X(12)=  93.6   Y(12)=      188.637
X(13)=  31.8   Y(13)=       63.876
X(14)=  55.3   Y(14)=      110.817
X(15)=  43.3   Y(15)=       86.795
X(16)=  46.4   Y(16)=       93.894
X(17)=  86.5   Y(17)=      174.043
X(18)=  54.4   Y(18)=      109.226
X(19)=  63.4   Y(19)=      127.126
X(20)=  72.3   Y(20)=      148.372
T(A0)=        .301 T(A1)=        -.311

A0=   1.0 EST A0=     .949 A1=   2.0 EST A1=   2.001
X( 1)=  95.0   Y( 1)=      193.227
X( 2)=  68.8   Y( 2)=      139.128
```

```
X( 3)=  86.2   Y( 3)=     172.423
X( 4)=  83.5   Y( 4)=     167.433
X( 5)=  59.1   Y( 5)=     119.797
X( 6)=  11.4   Y( 6)=      24.263
X( 7)=  53.0   Y( 7)=     106.436
X( 8)=  33.0   Y( 8)=      66.928
X( 9)=  20.8   Y( 9)=      42.676
X(10)=   4.2   Y(10)=       8.925
X(11)=  83.9   Y(11)=     168.201
X(12)=  93.6   Y(12)=     188.436
X(13)=  31.8   Y(13)=      66.175
X(14)=  55.3   Y(14)=     111.650
X(15)=  43.3   Y(15)=      86.744
X(16)=  46.4   Y(16)=      94.414
X(17)=  86.5   Y(17)=     174.905
X(18)=  54.4   Y(18)=     109.125
X(19)=  63.4   Y(19)=     126.868
X(20)=  72.3   Y(20)=     144.790
T(A0)=      -.105 T(A1)=       .203

A0=   1.0 EST A0=   1.315 A1=   2.0 EST A1=   1.993
X( 1)=  95.0   Y( 1)=     190.868
X( 2)=  68.8   Y( 2)=     140.091
X( 3)=  86.2   Y( 3)=     173.191
X( 4)=  83.5   Y( 4)=     167.329
X( 5)=  59.1   Y( 5)=     118.339
X( 6)=  11.4   Y( 6)=      22.834
X( 7)=  53.0   Y( 7)=     106.123
X( 8)=  33.0   Y( 8)=      67.339
X( 9)=  20.8   Y( 9)=      42.111
X(10)=   4.2   Y(10)=      11.120
X(11)=  83.9   Y(11)=     168.373
X(12)=  93.6   Y(12)=     187.584
X(13)=  31.8   Y(13)=      63.785
X(14)=  55.3   Y(14)=     112.039
X(15)=  43.3   Y(15)=      86.939
X(16)=  46.4   Y(16)=      95.995
X(17)=  86.5   Y(17)=     173.102
X(18)=  54.4   Y(18)=     109.505
X(19)=  63.4   Y(19)=     127.646
X(20)=  72.3   Y(20)=     145.993
T(A0)=       .638 T(A1)=      -.869

A0=   1.0 EST A0=   1.521 A1=   2.0 EST A1=   1.998
X( 1)=  95.0   Y( 1)=     191.284
X( 2)=  68.8   Y( 2)=     142.099
X( 3)=  86.2   Y( 3)=     172.766
X( 4)=  83.5   Y( 4)=     170.031
X( 5)=  59.1   Y( 5)=     118.827
X( 6)=  11.4   Y( 6)=      24.898
X( 7)=  53.0   Y( 7)=     107.010
X( 8)=  33.0   Y( 8)=      69.629
X( 9)=  20.8   Y( 9)=      42.579
```

```
X(10)=   4.2   Y(10)=        9.529
X(11)=  83.9   Y(11)=      169.511
X(12)=  93.6   Y(12)=      187.696
X(13)=  31.8   Y(13)=       64.075
X(14)=  55.3   Y(14)=      111.921
X(15)=  43.3   Y(15)=       86.871
X(16)=  46.4   Y(16)=       95.605
X(17)=  86.5   Y(17)=      174.547
X(18)=  54.4   Y(18)=      109.063
X(19)=  63.4   Y(19)=      127.298
X(20)=  72.3   Y(20)=      145.318
T(A0)=       .795 T(A1)=     -.140
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR EXPONENTIAL WITH HORIZONTAL REGRESSION

```
A0=   1.0 EST A0=    .514 A1=   0.0 EST A1=    .008
X( 1)=  66.6   Y( 1)=        1.558
X( 2)=  19.2   Y( 2)=         .220
X( 3)=  44.4   Y( 3)=        2.339
X( 4)=  64.2   Y( 4)=         .180
X( 5)=  70.5   Y( 5)=         .229
X( 6)=  84.0   Y( 6)=         .689
X( 7)=  46.7   Y( 7)=         .717
X( 8)=  26.7   Y( 8)=         .118
X( 9)=  93.7   Y( 9)=        1.317
X(10)=  49.5   Y(10)=        1.815
X(11)=  19.5   Y(11)=         .082
X(12)=  46.6   Y(12)=        1.293
X(13)=  74.3   Y(13)=        3.014
X(14)=   7.1   Y(14)=        1.431
X(15)=  33.3   Y(15)=         .879
X(16)=   4.2   Y(16)=         .781
X(17)=  66.4   Y(17)=         .385
X(18)=  85.5   Y(18)=        1.064
X(19)=  15.2   Y(19)=         .140
X(20)=   6.3   Y(20)=         .295
T(A0)=     -1.424 T(A1)=     1.416

A0=   1.0 EST A0=   1.338 A1=   0.0 EST A1=   -.003
X( 1)=  66.6   Y( 1)=         .197
X( 2)=  19.2   Y( 2)=        1.366
X( 3)=  44.4   Y( 3)=        2.769
X( 4)=  64.2   Y( 4)=        1.276
X( 5)=  70.5   Y( 5)=        1.374
X( 6)=  84.0   Y( 6)=         .101
X( 7)=  46.7   Y( 7)=         .423
X( 8)=  26.7   Y( 8)=        3.030
X( 9)=  93.7   Y( 9)=         .131
X(10)=  49.5   Y(10)=         .954
```

```
X(11)=  19.5   Y(11)=        1.690
X(12)=  46.6   Y(12)=         .130
X(13)=  74.3   Y(13)=         .337
X(14)=   7.1   Y(14)=         .047
X(15)=  33.3   Y(15)=        2.642
X(16)=   4.2   Y(16)=         .838
X(17)=  66.4   Y(17)=         .101
X(18)=  85.5   Y(18)=        3.731
X(19)=  15.2   Y(19)=         .487
X(20)=   6.3   Y(20)=        1.522
T(A0)=        .685 T(A1)=    -.427

A0=   1.0 EST A0=     .843 A1=   0.0 EST A1=     .003
X( 1)=  66.6   Y( 1)=        2.379
X( 2)=  19.2   Y( 2)=        3.279
X( 3)=  44.4   Y( 3)=         .323
X( 4)=  64.2   Y( 4)=        1.532
X( 5)=  70.5   Y( 5)=         .426
X( 6)=  84.0   Y( 6)=         .389
X( 7)=  46.7   Y( 7)=         .328
X( 8)=  26.7   Y( 8)=         .172
X( 9)=  93.7   Y( 9)=         .095
X(10)=  49.5   Y(10)=        1.317
X(11)=  19.5   Y(11)=        1.234
X(12)=  46.6   Y(12)=         .194
X(13)=  74.3   Y(13)=        3.191
X(14)=   7.1   Y(14)=        1.862
X(15)=  33.3   Y(15)=         .202
X(16)=   4.2   Y(16)=         .110
X(17)=  66.4   Y(17)=        2.231
X(18)=  85.5   Y(18)=         .644
X(19)=  15.2   Y(19)=         .224
X(20)=   6.3   Y(20)=         .048
T(A0)=       -.333 T(A1)=      .413

A0=   1.0 EST A0=     .828 A1=   0.0 EST A1=     .002
X( 1)=  66.6   Y( 1)=         .750
X( 2)=  19.2   Y( 2)=        1.054
X( 3)=  44.4   Y( 3)=        1.255
X( 4)=  64.2   Y( 4)=         .061
X( 5)=  70.5   Y( 5)=        1.837
X( 6)=  84.0   Y( 6)=         .026
X( 7)=  46.7   Y( 7)=         .330
X( 8)=  26.7   Y( 8)=        2.806
X( 9)=  93.7   Y( 9)=         .437
X(10)=  49.5   Y(10)=        1.736
X(11)=  19.5   Y(11)=         .126
X(12)=  46.6   Y(12)=         .361
X(13)=  74.3   Y(13)=        1.875
X(14)=   7.1   Y(14)=         .210
X(15)=  33.3   Y(15)=         .006
X(16)=   4.2   Y(16)=         .089
X(17)=  66.4   Y(17)=        3.811
```

```
X(18)=  85.5   Y(18)=          .117
X(19)=  15.2   Y(19)=         1.874
X(20)=   6.3   Y(20)=          .479
T(A0)=      -.364 T(A1)=       .333

A0=   1.0 EST A0=   1.307 A1=   0.0 EST A1=   -.005
X( 1)=  66.6   Y( 1)=         2.796
X( 2)=  19.2   Y( 2)=          .210
X( 3)=  44.4   Y( 3)=          .850
X( 4)=  64.2   Y( 4)=          .077
X( 5)=  70.5   Y( 5)=         3.551
X( 6)=  84.0   Y( 6)=          .737
X( 7)=  46.7   Y( 7)=         2.743
X( 8)=  26.7   Y( 8)=         1.678
X( 9)=  93.7   Y( 9)=          .003
X(10)=  49.5   Y(10)=          .030
X(11)=  19.5   Y(11)=         1.761
X(12)=  46.6   Y(12)=          .631
X(13)=  74.3   Y(13)=          .296
X(14)=   7.1   Y(14)=          .845
X(15)=  33.3   Y(15)=          .634
X(16)=   4.2   Y(16)=          .110
X(17)=  66.4   Y(17)=          .073
X(18)=  85.5   Y(18)=          .170
X(19)=  15.2   Y(19)=         3.201
X(20)=   6.3   Y(20)=          .988
T(A0)=        .606 T(A1)=     -.548
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR EXPONENTIAL WITH NEGATIVE REGRESSION

```
A0=   1.0 EST A0=   1.383 A1=  -2.0 EST A1=  -2.007
X( 1)=  27.3   Y( 1)=       -51.124
X( 2)=  66.4   Y( 2)=      -132.430
X( 3)=  22.7   Y( 3)=       -44.820
X( 4)=  14.4   Y( 4)=       -28.340
X( 5)=  30.0   Y( 5)=       -57.053
X( 6)=  73.3   Y( 6)=      -146.290
X( 7)=  99.2   Y( 7)=      -198.118
X( 8)=  11.0   Y( 8)=       -21.588
X( 9)=  50.3   Y( 9)=       -99.870
X(10)=  28.0   Y(10)=       -53.668
X(11)=  73.4   Y(11)=      -146.046
X(12)=  57.4   Y(12)=      -114.393
X(13)=  30.9   Y(13)=       -61.498
X(14)=  30.5   Y(14)=       -59.555
X(15)=  19.2   Y(15)=       -38.082
X(16)=  48.6   Y(16)=       -94.965
X(17)=  57.5   Y(17)=      -113.999
X(18)=  67.6   Y(18)=      -134.930
```

```
X(19)=   1.0   Y(19)=      -1.145
X(20)=  90.9   Y(20)=    -180.565
T(A0)=       .909 T(A1)=     -.961

A0=   1.0 EST A0=    1.037 A1=  -2.0 EST A1=  -2.002
X( 1)=  27.3   Y( 1)=     -50.929
X( 2)=  66.4   Y( 2)=    -132.015
X( 3)=  22.7   Y( 3)=     -44.327
X( 4)=  14.4   Y( 4)=     -27.788
X( 5)=  30.0   Y( 5)=     -59.295
X( 6)=  73.3   Y( 6)=    -145.203
X( 7)=  99.2   Y( 7)=    -196.415
X( 8)=  11.0   Y( 8)=     -20.830
X( 9)=  50.3   Y( 9)=    -100.207
X(10)=  28.0   Y(10)=     -54.535
X(11)=  73.4   Y(11)=    -145.751
X(12)=  57.4   Y(12)=    -114.756
X(13)=  30.9   Y(13)=     -61.270
X(14)=  30.5   Y(14)=     -60.588
X(15)=  19.2   Y(15)=     -37.484
X(16)=  48.6   Y(16)=     -97.169
X(17)=  57.5   Y(17)=    -114.020
X(18)=  67.6   Y(18)=    -135.154
X(19)=   1.0   Y(19)=      -1.466
X(20)=  90.9   Y(20)=    -181.641
T(A0)=       .099 T(A1)=     -.375

A0=   1.0 EST A0=     .894 A1=  -2.0 EST A1=  -2.000
X( 1)=  27.3   Y( 1)=     -53.385
X( 2)=  66.4   Y( 2)=    -132.420
X( 3)=  22.7   Y( 3)=     -42.966
X( 4)=  14.4   Y( 4)=     -28.752
X( 5)=  30.0   Y( 5)=     -59.689
X( 6)=  73.3   Y( 6)=    -146.061
X( 7)=  99.2   Y( 7)=    -198.362
X( 8)=  11.0   Y( 8)=     -21.214
X( 9)=  50.3   Y( 9)=     -98.913
X(10)=  28.0   Y(10)=     -55.397
X(11)=  73.4   Y(11)=    -144.924
X(12)=  57.4   Y(12)=    -113.848
X(13)=  30.9   Y(13)=     -60.505
X(14)=  30.5   Y(14)=     -60.540
X(15)=  19.2   Y(15)=     -37.657
X(16)=  48.6   Y(16)=     -96.705
X(17)=  57.5   Y(17)=    -114.699
X(18)=  67.6   Y(18)=    -132.175
X(19)=   1.0   Y(19)=      -1.538
X(20)=  90.9   Y(20)=    -181.617
T(A0)=      -.289 T(A1)=     -.011

A0=   1.0 EST A0=     .778 A1=  -2.0 EST A1=  -1.996
X( 1)=  27.3   Y( 1)=     -53.790
X( 2)=  66.4   Y( 2)=    -131.050
```

```
X( 3)=  22.7   Y( 3)=     -43.772
X( 4)=  14.4   Y( 4)=     -28.290
X( 5)=  30.0   Y( 5)=     -59.001
X( 6)=  73.3   Y( 6)=    -145.987
X( 7)=  99.2   Y( 7)=    -197.942
X( 8)=  11.0   Y( 8)=     -20.791
X( 9)=  50.3   Y( 9)=    -100.109
X(10)=  28.0   Y(10)=     -55.833
X(11)=  73.4   Y(11)=    -145.524
X(12)=  57.4   Y(12)=    -112.920
X(13)=  30.9   Y(13)=     -60.874
X(14)=  30.5   Y(14)=     -60.940
X(15)=  19.2   Y(15)=     -37.362
X(16)=  48.6   Y(16)=     -95.721
X(17)=  57.5   Y(17)=    -112.217
X(18)=  67.6   Y(18)=    -135.030
X(19)=   1.0   Y(19)=      -1.956
X(20)=  90.9   Y(20)=    -180.957
T(A0)=      -.711 T(A1)=      .665

A0=   1.0 EST A0=   2.119 A1=  -2.0 EST A1=  -2.014
X( 1)=  27.3   Y( 1)=     -51.937
X( 2)=  66.4   Y( 2)=    -132.480
X( 3)=  22.7   Y( 3)=     -42.716
X( 4)=  14.4   Y( 4)=     -28.118
X( 5)=  30.0   Y( 5)=     -52.870
X( 6)=  73.3   Y( 6)=    -146.519
X( 7)=  99.2   Y( 7)=    -197.793
X( 8)=  11.0   Y( 8)=     -18.761
X( 9)=  50.3   Y( 9)=    -100.012
X(10)=  28.0   Y(10)=     -55.925
X(11)=  73.4   Y(11)=    -144.747
X(12)=  57.4   Y(12)=    -114.016
X(13)=  30.9   Y(13)=     -61.211
X(14)=  30.5   Y(14)=     -60.074
X(15)=  19.2   Y(15)=     -37.570
X(16)=  48.6   Y(16)=     -93.450
X(17)=  57.5   Y(17)=    -114.938
X(18)=  67.6   Y(18)=    -133.252
X(19)=   1.0   Y(19)=      -1.967
X(20)=  90.9   Y(20)=    -181.632
T(A0)=      1.457 T(A1)=     -.999
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR CHI-SQUARE WITH POSITIVE REGRESSION

```
A0=   2.0 EST A0=   1.339 A1=   2.0 EST A1=   2.012
X( 1)=  95.0   Y( 1)=     192.306
X( 2)=  68.8   Y( 2)=     138.690
X( 3)=  86.2   Y( 3)=     174.387
```

```
X( 4)=  83.5   Y( 4)=     168.489
X( 5)=  59.1   Y( 5)=     123.200
X( 6)=  11.4   Y( 6)=      23.848
X( 7)=  53.0   Y( 7)=     109.032
X( 8)=  33.0   Y( 8)=      67.207
X( 9)=  20.8   Y( 9)=      42.977
X(10)=   4.2   Y(10)=       8.647
X(11)=  83.9   Y(11)=     172.301
X(12)=  93.6   Y(12)=     187.995
X(13)=  31.8   Y(13)=      65.523
X(14)=  55.3   Y(14)=     111.046
X(15)=  43.3   Y(15)=      92.008
X(16)=  46.4   Y(16)=      94.547
X(17)=  86.5   Y(17)=     177.436
X(18)=  54.4   Y(18)=     111.886
X(19)=  63.4   Y(19)=     126.886
X(20)=  72.3   Y(20)=     144.869
T(A0)=      -.748 T(A1)=     .917

A0=   2.0 EST A0=   1.181 A1=   2.0 EST A1=   2.012
X( 1)=  95.0   Y( 1)=     193.195
X( 2)=  68.8   Y( 2)=     139.178
X( 3)=  86.2   Y( 3)=     173.378
X( 4)=  83.5   Y( 4)=     168.915
X( 5)=  59.1   Y( 5)=     119.783
X( 6)=  11.4   Y( 6)=      23.346
X( 7)=  53.0   Y( 7)=     106.433
X( 8)=  33.0   Y( 8)=      66.701
X( 9)=  20.8   Y( 9)=      46.592
X(10)=   4.2   Y(10)=      10.529
X(11)=  83.9   Y(11)=     171.785
X(12)=  93.6   Y(12)=     192.285
X(13)=  31.8   Y(13)=      64.632
X(14)=  55.3   Y(14)=     113.490
X(15)=  43.3   Y(15)=      87.826
X(16)=  46.4   Y(16)=      93.959
X(17)=  86.5   Y(17)=     174.042
X(18)=  54.4   Y(18)=     109.507
X(19)=  63.4   Y(19)=     127.301
X(20)=  72.3   Y(20)=     147.197
T(A0)=     -1.068 T(A1)=    1.054

A0=   2.0 EST A0=   1.742 A1=   2.0 EST A1=   2.003
X( 1)=  95.0   Y( 1)=     193.032
X( 2)=  68.8   Y( 2)=     138.807
X( 3)=  86.2   Y( 3)=     173.778
X( 4)=  83.5   Y( 4)=     167.247
X( 5)=  59.1   Y( 5)=     122.701
X( 6)=  11.4   Y( 6)=      23.595
X( 7)=  53.0   Y( 7)=     107.923
X( 8)=  33.0   Y( 8)=      66.446
X( 9)=  20.8   Y( 9)=      47.008
X(10)=   4.2   Y(10)=      10.147
```

```
X(11)=  83.9   Y(11)=      172.236
X(12)=  93.6   Y(12)=      190.286
X(13)=  31.8   Y(13)=       63.686
X(14)=  55.3   Y(14)=      110.869
X(15)=  43.3   Y(15)=       89.795
X(16)=  46.4   Y(16)=       94.378
X(17)=  86.5   Y(17)=      173.978
X(18)=  54.4   Y(18)=      110.715
X(19)=  63.4   Y(19)=      128.383
X(20)=  72.3   Y(20)=      145.146
T(A0)=      -.304 T(A1)=      .229

A0=   2.0 EST A0=   1.458 A1=   2.0 EST A1=   2.006
X( 1)=  95.0   Y( 1)=      192.500
X( 2)=  68.8   Y( 2)=      139.709
X( 3)=  86.2   Y( 3)=      173.492
X( 4)=  83.5   Y( 4)=      167.399
X( 5)=  59.1   Y( 5)=      119.876
X( 6)=  11.4   Y( 6)=       24.421
X( 7)=  53.0   Y( 7)=      109.569
X( 8)=  33.0   Y( 8)=       66.519
X( 9)=  20.8   Y( 9)=       43.364
X(10)=   4.2   Y(10)=       10.029
X(11)=  83.9   Y(11)=      170.123
X(12)=  93.6   Y(12)=      191.030
X(13)=  31.8   Y(13)=       65.522
X(14)=  55.3   Y(14)=      113.440
X(15)=  43.3   Y(15)=       88.243
X(16)=  46.4   Y(16)=       94.155
X(17)=  86.5   Y(17)=      174.090
X(18)=  54.4   Y(18)=      110.010
X(19)=  63.4   Y(19)=      127.666
X(20)=  72.3   Y(20)=      146.960
T(A0)=     -1.110 T(A1)=      .808

A0=   2.0 EST A0=   1.885 A1=   2.0 EST A1=   2.000
X( 1)=  95.0   Y( 1)=      190.933
X( 2)=  68.8   Y( 2)=      137.718
X( 3)=  86.2   Y( 3)=      175.999
X( 4)=  83.5   Y( 4)=      167.217
X( 5)=  59.1   Y( 5)=      121.328
X( 6)=  11.4   Y( 6)=       24.285
X( 7)=  53.0   Y( 7)=      108.047
X( 8)=  33.0   Y( 8)=       69.132
X( 9)=  20.8   Y( 9)=       42.792
X(10)=   4.2   Y(10)=        9.051
X(11)=  83.9   Y(11)=      170.118
X(12)=  93.6   Y(12)=      189.440
X(13)=  31.8   Y(13)=       68.795
X(14)=  55.3   Y(14)=      112.147
X(15)=  43.3   Y(15)=       88.139
X(16)=  46.4   Y(16)=       93.791
X(17)=  86.5   Y(17)=      175.812
```

```
X(18)=  54.4   Y(18)=      110.744
X(19)=  63.4   Y(19)=      128.332
X(20)=  72.3   Y(20)=      146.382
T(A0)=      -.170 T(A1)=      .057
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR CHI-SQUARE WITH HORIZONTAL REGRESSION

```
A0=   2.0 EST A0=   1.998 A1=   0.0 EST A1=   -.007
X( 1)=  66.6   Y( 1)=        2.393
X( 2)=  19.2   Y( 2)=        1.768
X( 3)=  44.4   Y( 3)=        1.062
X( 4)=  64.2   Y( 4)=        1.749
X( 5)=  70.5   Y( 5)=        1.817
X( 6)=  84.0   Y( 6)=         .740
X( 7)=  46.7   Y( 7)=        2.434
X( 8)=  26.7   Y( 8)=        1.510
X( 9)=  93.7   Y( 9)=         .249
X(10)=  49.5   Y(10)=         .856
X(11)=  19.5   Y(11)=        1.840
X(12)=  46.6   Y(12)=        2.337
X(13)=  74.3   Y(13)=        2.545
X(14)=   7.1   Y(14)=         .818
X(15)=  33.3   Y(15)=        3.120
X(16)=   4.2   Y(16)=        2.703
X(17)=  66.4   Y(17)=         .492
X(18)=  85.5   Y(18)=        1.724
X(19)=  15.2   Y(19)=        2.049
X(20)=   6.3   Y(20)=        1.272
T(A0)=      -.003 T(A1)=    -1.115

A0=   2.0 EST A0=   2.745 A1=   0.0 EST A1=   -.016
X( 1)=  66.6   Y( 1)=        1.564
X( 2)=  19.2   Y( 2)=        1.338
X( 3)=  44.4   Y( 3)=         .644
X( 4)=  64.2   Y( 4)=        1.279
X( 5)=  70.5   Y( 5)=         .424
X( 6)=  84.0   Y( 6)=        3.664
X( 7)=  46.7   Y( 7)=        2.681
X( 8)=  26.7   Y( 8)=        2.830
X( 9)=  93.7   Y( 9)=         .048
X(10)=  49.5   Y(10)=        3.908
X(11)=  19.5   Y(11)=        3.908
X(12)=  46.6   Y(12)=        2.762
X(13)=  74.3   Y(13)=         .937
X(14)=   7.1   Y(14)=         .811
X(15)=  33.3   Y(15)=         .762
X(16)=   4.2   Y(16)=        2.283
X(17)=  66.4   Y(17)=        2.210
X(18)=  85.5   Y(18)=        1.227
```

```
X(19)=  15.2   Y(19)=        1.038
X(20)=   6.3   Y(20)=        5.670
T(A0)=      1.216 T(A1)=    -1.421

A0=   2.0 EST A0=   1.388 A1=   0.0 EST A1=     .014
X( 1)=  66.6   Y( 1)=        5.023
X( 2)=  19.2   Y( 2)=        2.885
X( 3)=  44.4   Y( 3)=         .233
X( 4)=  64.2   Y( 4)=        1.240
X( 5)=  70.5   Y( 5)=        2.978
X( 6)=  84.0   Y( 6)=        2.797
X( 7)=  46.7   Y( 7)=        1.245
X( 8)=  26.7   Y( 8)=        2.040
X( 9)=  93.7   Y( 9)=        2.257
X(10)=  49.5   Y(10)=        1.402
X(11)=  19.5   Y(11)=        1.181
X(12)=  46.6   Y(12)=        2.485
X(13)=  74.3   Y(13)=        4.230
X(14)=   7.1   Y(14)=        2.219
X(15)=  33.3   Y(15)=         .637
X(16)=   4.2   Y(16)=        3.000
X(17)=  66.4   Y(17)=        3.382
X(18)=  85.5   Y(18)=        1.036
X(19)=  15.2   Y(19)=         .417
X(20)=   6.3   Y(20)=         .749
T(A0)=     -1.126 T(A1)=     1.472

A0=   2.0 EST A0=   2.552 A1=   0.0 EST A1=    -.010
X( 1)=  66.6   Y( 1)=        2.129
X( 2)=  19.2   Y( 2)=        3.985
X( 3)=  44.4   Y( 3)=        5.085
X( 4)=  64.2   Y( 4)=        1.032
X( 5)=  70.5   Y( 5)=        2.890
X( 6)=  84.0   Y( 6)=        1.226
X( 7)=  46.7   Y( 7)=        1.159
X( 8)=  26.7   Y( 8)=        1.042
X( 9)=  93.7   Y( 9)=         .707
X(10)=  49.5   Y(10)=         .501
X(11)=  19.5   Y(11)=        2.597
X(12)=  46.6   Y(12)=        2.481
X(13)=  74.3   Y(13)=         .759
X(14)=   7.1   Y(14)=        4.979
X(15)=  33.3   Y(15)=        3.328
X(16)=   4.2   Y(16)=         .777
X(17)=  66.4   Y(17)=         .546
X(18)=  85.5   Y(18)=        3.799
X(19)=  15.2   Y(19)=        1.600
X(20)=   6.3   Y(20)=         .827
T(A0)=       .850 T(A1)=      -.862

A0=   2.0 EST A0=   3.324 A1=   0.0 EST A1=    -.014
X( 1)=  66.6   Y( 1)=        2.552
X( 2)=  19.2   Y( 2)=        6.516
```

```
X( 3)=  44.4   Y( 3)=        1.115
X( 4)=  64.2   Y( 4)=        4.787
X( 5)=  70.5   Y( 5)=        1.571
X( 6)=  84.0   Y( 6)=        3.630
X( 7)=  46.7   Y( 7)=        2.161
X( 8)=  26.7   Y( 8)=        5.501
X( 9)=  93.7   Y( 9)=        2.116
X(10)=  49.5   Y(10)=        2.333
X(11)=  19.5   Y(11)=        3.130
X(12)=  46.6   Y(12)=        1.351
X(13)=  74.3   Y(13)=        1.315
X(14)=   7.1   Y(14)=        2.602
X(15)=  33.3   Y(15)=         .928
X(16)=   4.2   Y(16)=        4.512
X(17)=  66.4   Y(17)=        2.911
X(18)=  85.5   Y(18)=         .911
X(19)=  15.2   Y(19)=        1.354
X(20)=   6.3   Y(20)=        1.719
T(A0)=      1.933 T(A1)=    -1.147
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR CHI-SQUARE WITH NEGATIVE REGRESSION

```
A0=   2.0 EST A0=   2.234 A1=  -2.0 EST A1=  -2.006
X( 1)=  27.3   Y( 1)=      -51.673
X( 2)=  66.4   Y( 2)=     -131.980
X( 3)=  22.7   Y( 3)=      -41.463
X( 4)=  14.4   Y( 4)=      -28.073
X( 5)=  30.0   Y( 5)=      -59.162
X( 6)=  73.3   Y( 6)=     -144.927
X( 7)=  99.2   Y( 7)=     -196.682
X( 8)=  11.0   Y( 8)=      -21.432
X( 9)=  50.3   Y( 9)=      -98.002
X(10)=  28.0   Y(10)=      -52.734
X(11)=  73.4   Y(11)=     -146.337
X(12)=  57.4   Y(12)=     -112.235
X(13)=  30.9   Y(13)=      -57.033
X(14)=  30.5   Y(14)=      -58.245
X(15)=  19.2   Y(15)=      -36.432
X(16)=  48.6   Y(16)=      -95.382
X(17)=  57.5   Y(17)=     -113.847
X(18)=  67.6   Y(18)=     -132.960
X(19)=   1.0   Y(19)=       -1.373
X(20)=  90.9   Y(20)=     -180.822
T(A0)=       .439 T(A1)=      -.686

A0=   2.0 EST A0=   2.540 A1=  -2.0 EST A1=  -2.007
X( 1)=  27.3   Y( 1)=      -53.833
X( 2)=  66.4   Y( 2)=     -130.134
X( 3)=  22.7   Y( 3)=      -40.931
```

```
X( 4)=  14.4   Y( 4)=     -26.258
X( 5)=  30.0   Y( 5)=     -57.323
X( 6)=  73.3   Y( 6)=    -146.079
X( 7)=  99.2   Y( 7)=    -197.178
X( 8)=  11.0   Y( 8)=     -17.213
X( 9)=  50.3   Y( 9)=     -99.996
X(10)=  28.0   Y(10)=     -53.920
X(11)=  73.4   Y(11)=    -143.698
X(12)=  57.4   Y(12)=    -114.198
X(13)=  30.9   Y(13)=     -60.739
X(14)=  30.5   Y(14)=     -60.659
X(15)=  19.2   Y(15)=     -34.087
X(16)=  48.6   Y(16)=     -95.294
X(17)=  57.5   Y(17)=    -114.480
X(18)=  67.6   Y(18)=    -129.578
X(19)=   1.0   Y(19)=       -.663
X(20)=  90.9   Y(20)=    -178.922
T(A0)=        .756 T(A1)=     -.554

A0=   2.0 EST A0=   1.495 A1=  -2.0 EST A1=  -1.986
X( 1)=  27.3   Y( 1)=     -53.525
X( 2)=  66.4   Y( 2)=    -129.594
X( 3)=  22.7   Y( 3)=     -45.269
X( 4)=  14.4   Y( 4)=     -27.367
X( 5)=  30.0   Y( 5)=     -58.346
X( 6)=  73.3   Y( 6)=    -143.017
X( 7)=  99.2   Y( 7)=    -195.786
X( 8)=  11.0   Y( 8)=     -20.917
X( 9)=  50.3   Y( 9)=     -99.784
X(10)=  28.0   Y(10)=     -55.146
X(11)=  73.4   Y(11)=    -144.667
X(12)=  57.4   Y(12)=    -113.475
X(13)=  30.9   Y(13)=     -58.902
X(14)=  30.5   Y(14)=     -58.074
X(15)=  19.2   Y(15)=     -34.849
X(16)=  48.6   Y(16)=     -94.099
X(17)=  57.5   Y(17)=    -113.271
X(18)=  67.6   Y(18)=    -133.839
X(19)=   1.0   Y(19)=        .736
X(20)=  90.9   Y(20)=    -178.029
T(A0)=      -1.102 T(A1)=      1.534

A0=   2.0 EST A0=   2.730 A1=  -2.0 EST A1=  -2.018
X( 1)=  27.3   Y( 1)=     -52.831
X( 2)=  66.4   Y( 2)=    -130.574
X( 3)=  22.7   Y( 3)=     -42.888
X( 4)=  14.4   Y( 4)=     -28.405
X( 5)=  30.0   Y( 5)=     -56.704
X( 6)=  73.3   Y( 6)=    -146.351
X( 7)=  99.2   Y( 7)=    -197.352
X( 8)=  11.0   Y( 8)=     -17.486
X( 9)=  50.3   Y( 9)=     -99.353
X(10)=  28.0   Y(10)=     -52.837
```

```
X(11)=  73.4   Y(11)=    -146.209
X(12)=  57.4   Y(12)=    -113.693
X(13)=  30.9   Y(13)=     -58.455
X(14)=  30.5   Y(14)=     -60.203
X(15)=  19.2   Y(15)=     -38.278
X(16)=  48.6   Y(16)=     -96.715
X(17)=  57.5   Y(17)=    -109.284
X(18)=  67.6   Y(18)=    -134.634
X(19)=   1.0   Y(19)=       1.343
X(20)=  90.9   Y(20)=    -180.478
T(A0)=      1.101 T(A1)=   -1.475

A0=   2.0 EST A0=   2.308 A1=  -2.0 EST A1=  -1.999
X( 1)=  27.3   Y( 1)=     -53.538
X( 2)=  66.4   Y( 2)=    -131.390
X( 3)=  22.7   Y( 3)=     -43.142
X( 4)=  14.4   Y( 4)=     -27.297
X( 5)=  30.0   Y( 5)=     -59.196
X( 6)=  73.3   Y( 6)=    -143.866
X( 7)=  99.2   Y( 7)=    -196.432
X( 8)=  11.0   Y( 8)=     -19.620
X( 9)=  50.3   Y( 9)=     -93.280
X(10)=  28.0   Y(10)=     -53.762
X(11)=  73.4   Y(11)=    -143.080
X(12)=  57.4   Y(12)=    -112.419
X(13)=  30.9   Y(13)=     -58.225
X(14)=  30.5   Y(14)=     -59.481
X(15)=  19.2   Y(15)=     -33.858
X(16)=  48.6   Y(16)=     -96.436
X(17)=  57.5   Y(17)=    -111.302
X(18)=  67.6   Y(18)=    -134.035
X(19)=   1.0   Y(19)=      -1.032
X(20)=  90.9   Y(20)=    -181.430
T(A0)=       .420 T(A1)=      .015
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR SKEWED NORMAL WITH POSITIVE REGRESSION

```
A0=   4.0 EST A0=   2.961 A1=   2.0 EST A1=   2.018
X( 1)=  95.0   Y( 1)=     194.578
X( 2)=  68.8   Y( 2)=     140.351
X( 3)=  86.2   Y( 3)=     176.525
X( 4)=  83.5   Y( 4)=     170.386
X( 5)=  59.1   Y( 5)=     126.270
X( 6)=  11.4   Y( 6)=      25.482
X( 7)=  53.0   Y( 7)=     111.566
X( 8)=  33.0   Y( 8)=      68.943
X( 9)=  20.8   Y( 9)=      44.813
X(10)=   4.2   Y(10)=       9.502
X(11)=  83.9   Y(11)=     175.252
```

```
X(12)=  93.6   Y(12)=      189.444
X(13)=  31.8   Y(13)=       67.632
X(14)=  55.3   Y(14)=      112.168
X(15)=  43.3   Y(15)=       95.169
X(16)=  46.4   Y(16)=       96.575
X(17)=  86.5   Y(17)=      180.371
X(18)=  54.4   Y(18)=      114.437
X(19)=  63.4   Y(19)=      127.407
X(20)=  72.3   Y(20)=      145.759
T(A0)=      -.813 T(A1)=      .932

A0=   4.0 EST A0=    2.791 A1=    2.0 EST A1=    2.018
X( 1)=  95.0   Y( 1)=      195.781
X( 2)=  68.8   Y( 2)=      141.122
X( 3)=  86.2   Y( 3)=      174.964
X( 4)=  83.5   Y( 4)=      171.020
X( 5)=  59.1   Y( 5)=      121.729
X( 6)=  11.4   Y( 6)=       24.573
X( 7)=  53.0   Y( 7)=      107.539
X( 8)=  33.0   Y( 8)=       68.073
X( 9)=  20.8   Y( 9)=       49.660
X(10)=   4.2   Y(10)=       12.728
X(11)=  83.9   Y(11)=      174.603
X(12)=  93.6   Y(12)=      195.374
X(13)=  31.8   Y(13)=       66.256
X(14)=  55.3   Y(14)=      115.977
X(15)=  43.3   Y(15)=       89.574
X(16)=  46.4   Y(16)=       95.666
X(17)=  86.5   Y(17)=      175.671
X(18)=  54.4   Y(18)=      110.884
X(19)=  63.4   Y(19)=      128.482
X(20)=  72.3   Y(20)=      149.579
T(A0)=     -1.113 T(A1)=     1.098

A0=   4.0 EST A0=    3.341 A1=    2.0 EST A1=    2.008
X( 1)=  95.0   Y( 1)=      194.846
X( 2)=  68.8   Y( 2)=      141.899
X( 3)=  86.2   Y( 3)=      175.155
X( 4)=  83.5   Y( 4)=      168.465
X( 5)=  59.1   Y( 5)=      121.869
X( 6)=  11.4   Y( 6)=       26.387
X( 7)=  53.0   Y( 7)=      112.269
X( 8)=  33.0   Y( 8)=       67.720
X( 9)=  20.8   Y( 9)=       45.399
X(10)=   4.2   Y(10)=       11.998
X(11)=  83.9   Y(11)=      172.401
X(12)=  93.6   Y(12)=      193.804
X(13)=  31.8   Y(13)=       67.631
X(14)=  55.3   Y(14)=      115.908
Y(15)=  43.3   Y(15)=       90.219
X(16)=  46.4   Y(16)=       95.978
X(17)=  86.5   Y(17)=      175.752
X(18)=  54.4   Y(18)=      111.747
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```
X(19)=  63.4   Y(19)=      129.171
X(20)=  72.3   Y(20)=      149.253
T(A0)=       -.908 T(A1)=      .703

A0=   4.0 EST A0=    3.862 A1=   2.0 EST A1=   2.000
X( 1)=  95.0   Y( 1)=      192.488
X( 2)=  68.8   Y( 2)=      138.323
X( 3)=  86.2   Y( 3)=      178.708
X( 4)=  83.5   Y( 4)=      168.023
X( 5)=  59.1   Y( 5)=      123.893
X( 6)=  11.4   Y( 6)=       26.180
X( 7)=  53.0   Y( 7)=      110.211
X( 8)=  33.0   Y( 8)=       71.698
X( 9)=  20.8   Y( 9)=       44.519
X(10)=   4.2   Y(10)=       10.379
X(11)=  83.9   Y(11)=      172.394
X(12)=  93.6   Y(12)=      191.685
X(13)=  31.8   Y(13)=       71.909
X(14)=  55.3   Y(14)=      114.074
X(15)=  43.3   Y(15)=       90.062
X(16)=  46.4   Y(16)=       95.386
X(17)=  86.5   Y(17)=      178.271
X(18)=  54.4   Y(18)=      112.863
X(19)=  63.4   Y(19)=      130.251
X(20)=  72.3   Y(20)=      148.426
T(A0)=       -.138 T(A1)=      .038

A0=   4.0 EST A0=    2.966 A1=   2.0 EST A1=   2.010
X( 1)=  95.0   Y( 1)=      194.699
X( 2)=  68.8   Y( 2)=      141.405
X( 3)=  86.2   Y( 3)=      175.105
X( 4)=  83.5   Y( 4)=      170.777
X( 5)=  59.1   Y( 5)=      122.077
X( 6)=  11.4   Y( 6)=       24.945
X( 7)=  53.0   Y( 7)=      110.755
X( 8)=  33.0   Y( 8)=       69.418
X( 9)=  20.8   Y( 9)=       42.708
X(10)=   4.2   Y(10)=       10.753
X(11)=  83.9   Y(11)=      171.711
X(12)=  93.6   Y(12)=      191.820
X(13)=  31.8   Y(13)=       68.508
X(14)=  55.3   Y(14)=      112.885
X(15)=  43.3   Y(15)=       92.282
X(16)=  46.4   Y(16)=       97.924
X(17)=  86.5   Y(17)=      174.664
X(18)=  54.4   Y(18)=      112.540
X(19)=  63.4   Y(19)=      131.014
X(20)=  72.3   Y(20)=      147.646
T(A0)=      -1.573 T(A1)=     1.050
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)

```
FOR SKEWED NORMAL WITH HORIZONTAL REGRESSION

AO=    4.0 EST AO=    3.804 A1=    0.0 EST A1=     .011
X( 1)=  66.6    Y( 1)=         1.966
X( 2)=  19.2    Y( 2)=         3.058
X( 3)=  44.4    Y( 3)=         1.521
X( 4)=  64.2    Y( 4)=         6.392
X( 5)=  70.5    Y( 5)=         5.094
X( 6)=  84.0    Y( 6)=         5.295
X( 7)=  46.7    Y( 7)=          .444
X( 8)=  26.7    Y( 8)=         6.705
X( 9)=  93.7    Y( 9)=         6.704
X(10)=  49.5    Y(10)=         5.203
X(11)=  19.5    Y(11)=         2.494
X(12)=  46.6    Y(12)=         2.273
X(13)=  74.3    Y(13)=         2.186
X(14)=   7.1    Y(14)=         4.545
X(15)=  33.3    Y(15)=         4.442
X(16)=   4.2    Y(16)=         2.975
X(17)=  66.4    Y(17)=         2.665
X(18)=  85.5    Y(18)=         8.886
X(19)=  15.2    Y(19)=         8.098
X(20)=   6.3    Y(20)=         5.369
T(AO)=       -.193 T(A1)=       .590

AO=    4.0 EST AO=    5.367 A1=    0.0 EST A1=    -.028
X( 1)=  66.6    Y( 1)=         3.899
X( 2)=  19.2    Y( 2)=         2.676
X( 3)=  44.4    Y( 3)=         1.928
X( 4)=  64.2    Y( 4)=         1.204
X( 5)=  70.5    Y( 5)=         1.847
X( 6)=  84.0    Y( 6)=         5.021
X( 7)=  46.7    Y( 7)=         3.212
X( 8)=  26.7    Y( 8)=         7.335
X( 9)=  93.7    Y( 9)=         3.377
X(10)=  49.5    Y(10)=         2.850
X(11)=  19.5    Y(11)=         2.143
X(12)=  46.6    Y(12)=         5.207
X(13)=  74.3    Y(13)=         2.708
X(14)=   7.1    Y(14)=         8.050
X(15)=  33.3    Y(15)=         1.729
X(16)=   4.2    Y(16)=         3.700
X(17)=  66.4    Y(17)=         4.022
X(18)=  85.5    Y(18)=         5.122
X(19)=  15.2    Y(19)=         4.918
X(20)=   6.3    Y(20)=         9.899
T(AO)=       1.453 T(A1)=     -1.645

AO=    4.0 EST AO=    4.421 A1=    0.0 EST A1=     .003
X( 1)=  66.6    Y( 1)=         2.794
X( 2)=  19.2    Y( 2)=         7.807
X( 3)=  44.4    Y( 3)=         3.511
```

```
X( 4)=  64.2   Y( 4)=        6.349
X( 5)=  70.5   Y( 5)=        4.373
X( 6)=  84.0   Y( 6)=        8.681
X( 7)=  46.7   Y( 7)=        4.310
X( 8)=  26.7   Y( 8)=        4.615
X( 9)=  93.7   Y( 9)=        5.695
X(10)=  49.5   Y(10)=        3.172
X(11)=  19.5   Y(11)=        3.115
X(12)=  46.6   Y(12)=        4.987
X(13)=  74.3   Y(13)=        2.479
X(14)=   7.1   Y(14)=        7.466
X(15)=  33.3   Y(15)=        5.404
X(16)=   4.2   Y(16)=        2.449
X(17)=  66.4   Y(17)=        3.177
X(18)=  85.5   Y(18)=        3.733
X(19)=  15.2   Y(19)=        5.424
X(20)=   6.3   Y(20)=        2.287
T(A0)=        .506 T(A1)=      .239

A0=   4.0 EST A0=    4.826 A1=   0.0 EST A1=    -.020
X( 1)=  66.6   Y( 1)=        6.740
X( 2)=  19.2   Y( 2)=        2.120
X( 3)=  44.4   Y( 3)=        2.320
X( 4)=  64.2   Y( 4)=        3.663
X( 5)=  70.5   Y( 5)=        3.730
X( 6)=  84.0   Y( 6)=        1.816
X( 7)=  46.7   Y( 7)=        4.980
X( 8)=  26.7   Y( 8)=        5.874
X( 9)=  93.7   Y( 9)=        1.602
X(10)=  49.5   Y(10)=        4.934
X(11)=  19.5   Y(11)=        7.781
X(12)=  46.6   Y(12)=        5.192
X(13)=  74.3   Y(13)=        4.095
X(14)=   7.1   Y(14)=        3.878
X(15)=  33.3   Y(15)=        2.854
X(16)=   4.2   Y(16)=        4.484
X(17)=  66.4   Y(17)=        1.932
X(18)=  85.5   Y(18)=        2.563
X(19)=  15.2   Y(19)=        2.192
X(20)=   6.3   Y(20)=        5.072
T(A0)=       1.132 T(A1)=    -1.497

A0=   4.0 EST A0=    3.902 A1=   0.0 EST A1=     .009
X( 1)=  66.6   Y( 1)=        7.411
X( 2)=  19.2   Y( 2)=        4.902
X( 3)=  44.4   Y( 3)=        5.086
X( 4)=  64.2   Y( 4)=        1.722
X( 5)=  70.5   Y( 5)=        2.966
X( 6)=  84.0   Y( 6)=        7.806
X( 7)=  46.7   Y( 7)=        1.886
X( 8)=  26.7   Y( 8)=        4.257
X( 9)=  93.7   Y( 9)=        5.657
X(10)=  49.5   Y(10)=        1.883
```

```
X(11)=  19.5   Y(11)=       2.701
X(12)=  46.6   Y(12)=       1.332
X(13)=  74.3   Y(13)=       7.215
X(14)=   7.1   Y(14)=       4.006
X(15)=  33.3   Y(15)=       1.719
X(16)=   4.2   Y(16)=       8.828
X(17)=  66.4   Y(17)=       3.148
X(18)=  85.5   Y(18)=       5.359
X(19)=  15.2   Y(19)=       2.726
X(20)=   6.3   Y(20)=       5.794
T(A0)=      -.096 T(A1)=      .484
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR SKEWED NORMAL WITH NEGATIVE REGRESSION

```
A0=   4.0 EST A0=   5.083 A1=  -2.0 EST A1=  -2.021
X( 1)=  27.3   Y( 1)=     -49.598
X( 2)=  66.4   Y( 2)=    -130.061
X( 3)=  22.7   Y( 3)=     -43.118
X( 4)=  14.4   Y( 4)=     -26.451
X( 5)=  30.0   Y( 5)=     -55.667
X( 6)=  73.3   Y( 6)=    -143.470
X( 7)=  99.2   Y( 7)=    -193.014
X( 8)=  11.0   Y( 8)=     -16.576
X( 9)=  50.3   Y( 9)=     -94.354
X(10)=  28.0   Y(10)=     -50.343
X(11)=  73.4   Y(11)=    -143.053
X(12)=  57.4   Y(12)=    -111.612
X(13)=  30.9   Y(13)=     -56.630
X(14)=  30.5   Y(14)=     -54.472
X(15)=  19.2   Y(15)=     -34.594
X(16)=  48.6   Y(16)=     -92.735
X(17)=  57.5   Y(17)=    -110.137
X(18)=  67.6   Y(18)=    -133.746
X(19)=   1.0   Y(19)=       3.912
X(20)=  90.9   Y(20)=    -180.694
T(A0)=      1.623 T(A1)=    -1.648

A0=   4.0 EST A0=   2.922 A1=  -2.0 EST A1=  -2.012
X( 1)=  27.3   Y( 1)=     -54.045
X( 2)=  66.4   Y( 2)=    -130.389
X( 3)=  22.7   Y( 3)=     -39.423
X( 4)=  14.4   Y( 4)=     -26.571
X( 5)=  30.0   Y( 5)=     -57.573
X( 6)=  73.3   Y( 6)=    -146.226
X( 7)=  99.2   Y( 7)=    -197.494
X( 8)=  11.0   Y( 8)=     -15.497
X( 9)=  50.3   Y( 9)=    -100.166
X(10)=  28.0   Y(10)=     -54.324
X(11)=  73.4   Y(11)=    -143.590
```

```
X(12)=  57.4   Y(12)=    -114.367
X(13)=  30.9   Y(13)=     -61.021
X(14)=  30.5   Y(14)=     -60.756
X(15)=  19.2   Y(15)=     -32.695
X(16)=  48.6   Y(16)=     -95.699
X(17)=  57.5   Y(17)=    -114.627
X(18)=  67.6   Y(18)=    -127.430
X(19)=   1.0   Y(19)=      -1.002
X(20)=  90.9   Y(20)=    -179.033
T(A0)=     -1.046 T(A1)=      -.633

A0=   4.0 EST A0=   3.100 A1=  -2.0 EST A1=  -2.024
X( 1)=  27.3   Y( 1)=     -53.228
X( 2)=  66.4   Y( 2)=    -130.966
X( 3)=  22.7   Y( 3)=     -43.212
X( 4)=  14.4   Y( 4)=     -28.518
X( 5)=  30.0   Y( 5)=     -56.364
X( 6)=  73.3   Y( 6)=    -146.423
X( 7)=  99.2   Y( 7)=    -197.631
X( 8)=  11.0   Y( 8)=     -15.947
X( 9)=  50.3   Y( 9)=     -99.674
X(10)=  28.0   Y(10)=     -52.660
X(11)=  73.4   Y(11)=    -146.375
X(12)=  57.4   Y(12)=    -113.985
X(13)=  30.9   Y(13)=     -58.053
X(14)=  30.5   Y(14)=     -60.422
X(15)=  19.2   Y(15)=     -38.314
X(16)=  48.6   Y(16)=     -96.853
X(17)=  57.5   Y(17)=    -107.093
X(18)=  67.6   Y(18)=    -134.793
X(19)=   1.0   Y(19)=       1.743
X(20)=  90.9   Y(20)=    -180.814
T(A0)=      -.988 T(A1)=     -1.433

A0=   4.0 EST A0=   2.379 A1=  -2.0 EST A1=  -1.999
X( 1)=  27.3   Y( 1)=     -53.820
X( 2)=  66.4   Y( 2)=    -131.742
X( 3)=  22.7   Y( 3)=     -43.529
X( 4)=  14.4   Y( 4)=     -27.663
X( 5)=  30.0   Y( 5)=     -59.417
X( 6)=  73.3   Y( 6)=    -144.082
X( 7)=  99.2   Y( 7)=    -196.838
X( 8)=  11.0   Y( 8)=     -19.983
X( 9)=  50.3   Y( 9)=     -90.526
X(10)=  28.0   Y(10)=     -54.152
X(11)=  73.4   Y(11)=    -142.233
X(12)=  57.4   Y(12)=    -112.782
X(13)=  30.9   Y(13)=     -57.543
X(14)=  30.5   Y(14)=     -59.849
X(15)=  19.2   Y(15)=     -32.301
X(16)=  48.6   Y(16)=     -96.647
X(17)=  57.5   Y(17)=    -110.480
X(18)=  67.6   Y(18)=    -134.340
```

```
X(19)=   1.0   Y(19)=      -1.293
X(20)=  90.9   Y(20)=    -181.536
T(A0)=     -1.495 T(A1)=     .045

A0=   4.0 EST A0=    5.546 A1=  -2.0 EST A1=  -2.016
X( 1)=  27.3   Y( 1)=     -50.503
X( 2)=  66.4   Y( 2)=    -128.120
X( 3)=  22.7   Y( 3)=     -34.550
X( 4)=  14.4   Y( 4)=     -24.318
X( 5)=  30.0   Y( 5)=     -53.537
X( 6)=  73.3   Y( 6)=    -141.918
X( 7)=  99.2   Y( 7)=    -192.124
X( 8)=  11.0   Y( 8)=     -18.569
X( 9)=  50.3   Y( 9)=     -93.098
X(10)=  28.0   Y(10)=     -53.813
X(11)=  73.4   Y(11)=    -140.365
X(12)=  57.4   Y(12)=    -111.926
X(13)=  30.9   Y(13)=     -59.254
X(14)=  30.5   Y(14)=     -59.601
X(15)=  19.2   Y(15)=     -29.939
X(16)=  48.6   Y(16)=     -94.397
X(17)=  57.5   Y(17)=    -111.607
X(18)=  67.6   Y(18)=    -132.129
X(19)=   1.0   Y(19)=       5.680
X(20)=  90.9   Y(20)=    -179.190
T(A0)=      1.416 T(A1)=     -.800
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=1) WITH POSITIVE REGRESSION

```
A0=   1.0 EST A0=     .889 A1=   2.0 EST A1=   1.998
X( 1)=  95.0   Y( 1)=     189.609
X( 2)=  68.8   Y( 2)=     138.052
X( 3)=  86.2   Y( 3)=     171.730
X( 4)=  83.5   Y( 4)=     168.083
X( 5)=  59.1   Y( 5)=     119.751
X( 6)=  11.4   Y( 6)=      23.375
X( 7)=  53.0   Y( 7)=     106.993
X( 8)=  33.0   Y( 8)=      68.647
X( 9)=  20.8   Y( 9)=      42.976
X(10)=   4.2   Y(10)=       8.421
X(11)=  83.9   Y(11)=     168.220
X(12)=  93.6   Y(12)=     187.615
X(13)=  31.8   Y(13)=      63.444
X(14)=  55.3   Y(14)=     111.150
X(15)=  43.3   Y(15)=      86.970
X(16)=  46.4   Y(16)=      93.945
X(17)=  86.5   Y(17)=     175.716
X(18)=  54.4   Y(18)=     108.722
```

```
X(19)=  63.4   Y(19)=      128.403
X(20)=  72.3   Y(20)=      146.199
T(A0)=      -.218 T(A1)=      -.171

A0=   1.0 EST A0=      .499 A1=   2.0 EST A1=   1.999
X( 1)=  95.0   Y( 1)=      190.360
X( 2)=  68.8   Y( 2)=      139.371
X( 3)=  86.2   Y( 3)=      172.217
X( 4)=  83.5   Y( 4)=      168.338
X( 5)=  59.1   Y( 5)=      118.374
X( 6)=  11.4   Y( 6)=       23.966
X( 7)=  53.0   Y( 7)=      108.152
X( 8)=  33.0   Y( 8)=       65.974
X( 9)=  20.8   Y( 9)=       40.071
X(10)=   4.2   Y(10)=        9.884
X(11)=  83.9   Y(11)=      169.651
X(12)=  93.6   Y(12)=      187.614
X(13)=  31.8   Y(13)=       64.612
X(14)=  55.3   Y(14)=      110.085
X(15)=  43.3   Y(15)=       86.273
X(16)=  46.4   Y(16)=       93.216
X(17)=  86.5   Y(17)=      171.555
X(18)=  54.4   Y(18)=      108.729
X(19)=  63.4   Y(19)=      127.178
X(20)=  72.3   Y(20)=      145.342
T(A0)=      -.905 T(A1)=      -.081

A0=   1.0 EST A0=      .880 A1=   2.0 EST A1=   2.004
X( 1)=  95.0   Y( 1)=      192.862
X( 2)=  68.8   Y( 2)=      139.176
X( 3)=  86.2   Y( 3)=      172.126
X( 4)=  83.5   Y( 4)=      168.151
X( 5)=  59.1   Y( 5)=      119.891
X( 6)=  11.4   Y( 6)=       23.987
X( 7)=  53.0   Y( 7)=      108.477
X( 8)=  33.0   Y( 8)=       67.403
X( 9)=  20.8   Y( 9)=       41.404
X(10)=   4.2   Y(10)=        8.921
X(11)=  83.9   Y(11)=      168.192
X(12)=  93.6   Y(12)=      189.459
X(13)=  31.8   Y(13)=       63.561
X(14)=  55.3   Y(14)=      111.938
X(15)=  43.3   Y(15)=       87.830
X(16)=  46.4   Y(16)=       95.277
X(17)=  86.5   Y(17)=      171.920
X(18)=  54.4   Y(18)=      110.198
X(19)=  63.4   Y(19)=      128.551
X(20)=  72.3   Y(20)=      145.419
T(A0)=      -.214 T(A1)=       .528

A0=   1.0 EST A0=    1.139 A1=   2.0 EST A1=   1.999
X( 1)=  95.0   Y( 1)=      191.443
X( 2)=  68.8   Y( 2)=      139.061
```

```
X( 3)=   86.2   Y( 3)=       172.115
X( 4)=   83.5   Y( 4)=       168.044
X( 5)=   59.1   Y( 5)=       119.489
X( 6)=   11.4   Y( 6)=        23.088
X( 7)=   53.0   Y( 7)=       106.883
X( 8)=   33.0   Y( 8)=        68.912
X( 9)=   20.8   Y( 9)=        41.818
X(10)=    4.2   Y(10)=         9.038
X(11)=   83.9   Y(11)=       168.813
X(12)=   93.6   Y(12)=       188.384
X(13)=   31.8   Y(13)=        66.590
X(14)=   55.3   Y(14)=       111.871
X(15)=   43.3   Y(15)=        87.467
X(16)=   46.4   Y(16)=        93.419
X(17)=   86.5   Y(17)=       174.365
X(18)=   54.4   Y(18)=       109.747
X(19)=   63.4   Y(19)=       128.004
X(20)=   72.3   Y(20)=       145.576
T(A0)=        .331 T(A1)=       -.058

A0=   1.0 EST A0=    1.154 A1=   2.0 EST A1=    1.996
X( 1)=   95.0   Y( 1)=       190.104
X( 2)=   68.8   Y( 2)=       139.026
X( 3)=   86.2   Y( 3)=       173.184
X( 4)=   83.5   Y( 4)=       167.017
X( 5)=   59.1   Y( 5)=       119.166
X( 6)=   11.4   Y( 6)=        24.269
X( 7)=   53.0   Y( 7)=       106.368
X( 8)=   33.0   Y( 8)=        66.502
X( 9)=   20.8   Y( 9)=        43.311
X(10)=    4.2   Y(10)=         9.235
X(11)=   83.9   Y(11)=       169.514
X(12)=   93.6   Y(12)=       188.389
X(13)=   31.8   Y(13)=        63.699
X(14)=   55.3   Y(14)=       111.884
X(15)=   43.3   Y(15)=        88.184
X(16)=   46.4   Y(16)=        93.640
X(17)=   86.5   Y(17)=       173.890
X(18)=   54.4   Y(18)=       110.376
X(19)=   63.4   Y(19)=       127.537
X(20)=   72.3   Y(20)=       145.813
T(A0)=        .525 T(A1)=       -.706
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=1) WITH HORIZONTAL REGRESSION

```
A0=   1.0 EST A0=    1.237 A1=   0.0 EST A1=    -.002
X( 1)=   66.6   Y( 1)=         1.978
X( 2)=   19.2   Y( 2)=         2.253
X( 3)=   44.4   Y( 3)=          .963
```

```
X( 4)=   64.2   Y( 4)=        1.948
X( 5)=   70.5   Y( 5)=        2.048
X( 6)=   84.0   Y( 6)=        1.104
X( 7)=   46.7   Y( 7)=        -.045
X( 8)=   26.7   Y( 8)=        2.440
X( 9)=   93.7   Y( 9)=         .401
X(10)=   49.5   Y(10)=         .677
X(11)=   19.5   Y(11)=        1.109
X(12)=   46.6   Y(12)=        -.463
X(13)=   74.3   Y(13)=        2.797
X(14)=    7.1   Y(14)=         .734
X(15)=   33.3   Y(15)=        1.187
X(16)=    4.2   Y(16)=        2.994
X(17)=   66.4   Y(17)=       -1.002
X(18)=   85.5   Y(18)=        1.035
X(19)=   15.2   Y(19)=         .948
X(20)=    6.3   Y(20)=        -.423
T(A0)=         .484 T(A1)=    -.245

A0=   1.0 EST A0=     .900 A1=   0.0 EST A1=     .007
X( 1)=   66.6   Y( 1)=         .666
X( 2)=   19.2   Y( 2)=        1.597
X( 3)=   44.4   Y( 3)=         .163
X( 4)=   64.2   Y( 4)=        1.204
X( 5)=   70.5   Y( 5)=        1.560
X( 6)=   84.0   Y( 6)=        3.071
X( 7)=   46.7   Y( 7)=         .578
X( 8)=   26.7   Y( 8)=        1.920
X( 9)=   93.7   Y( 9)=        -.062
X(10)=   49.5   Y(10)=         .469
X(11)=   19.5   Y(11)=        1.357
X(12)=   46.6   Y(12)=        2.439
X(13)=   74.3   Y(13)=        2.557
X(14)=    7.1   Y(14)=         .550
X(15)=   33.3   Y(15)=        1.258
X(16)=    4.2   Y(16)=        1.522
X(17)=   66.4   Y(17)=         .181
X(18)=   85.5   Y(18)=        3.074
X(19)=   15.2   Y(19)=        -.956
X(20)=    6.3   Y(20)=        1.928
T(A0)=       -.214 T(A1)=      .888

A0=   1.0 EST A0=    1.183 A1=   0.0 EST A1=    -.004
X( 1)=   66.6   Y( 1)=         .567
X( 2)=   19.2   Y( 2)=        -.197
X( 3)=   44.4   Y( 3)=        1.472
X( 4)=   64.2   Y( 4)=        1.417
X( 5)=   70.5   Y( 5)=        -.522
X( 6)=   84.0   Y( 6)=        1.492
X( 7)=   46.7   Y( 7)=        2.303
X( 8)=   26.7   Y( 8)=         .749
X( 9)=   93.7   Y( 9)=         .670
X(10)=   49.5   Y(10)=         .906
```

```
X(11)=  19.5   Y(11)=       2.298
X(12)=  46.6   Y(12)=        .684
X(13)=  74.3   Y(13)=        .906
X(14)=   7.1   Y(14)=       -.196
X(15)=  33.3   Y(15)=       1.216
X(16)=   4.2   Y(16)=        .983
X(17)=  66.4   Y(17)=       1.946
X(18)=  85.5   Y(18)=       -.055
X(19)=  15.2   Y(19)=        .817
X(20)=   6.3   Y(20)=       2.405
T(A0)=         .489 T(A1)=      -.593

A0=   1.0 EST A0=     .823 A1=   0.0 EST A1=   0.000
X( 1)=  66.6   Y( 1)=       -.451
X( 2)=  19.2   Y( 2)=       1.087
X( 3)=  44.4   Y( 3)=        .861
X( 4)=  64.2   Y( 4)=        .710
X( 5)=  70.5   Y( 5)=       1.474
X( 6)=  84.0   Y( 6)=       1.994
X( 7)=  46.7   Y( 7)=       2.108
X( 8)=  26.7   Y( 8)=       1.658
X( 9)=  93.7   Y( 9)=        .483
X(10)=  49.5   Y(10)=       2.423
X(11)=  19.5   Y(11)=       1.319
X(12)=  46.6   Y(12)=       1.009
X(13)=  74.3   Y(13)=       -.664
X(14)=   7.1   Y(14)=        .136
X(15)=  33.3   Y(15)=       -.746
X(16)=   4.2   Y(16)=        .524
X(17)=  66.4   Y(17)=        .791
X(18)=  85.5   Y(18)=        .893
X(19)=  15.2   Y(19)=       1.670
X(20)=   6.3   Y(20)=       -.037
T(A0)=       -.441 T(A1)=        .114

A0=   1.0 EST A0=    1.039 A1=   0.0 EST A1=   -.004
X( 1)=  66.6   Y( 1)=       -.390
X( 2)=  19.2   Y( 2)=        .452
X( 3)=  44.4   Y( 3)=       -.669
X( 4)=  64.2   Y( 4)=       1.083
X( 5)=  70.5   Y( 5)=       1.551
X( 6)=  84.0   Y( 6)=        .575
X( 7)=  46.7   Y( 7)=        .993
X( 8)=  26.7   Y( 8)=       2.647
X( 9)=  93.7   Y( 9)=       1.376
X(10)=  49.5   Y(10)=        .021
X(11)=  19.5   Y(11)=        .420
X(12)=  46.6   Y(12)=        .415
X(13)=  74.3   Y(13)=       -.155
X(14)=   7.1   Y(14)=        .550
X(15)=  33.3   Y(15)=        .370
X(16)=   4.2   Y(16)=       1.145
X(17)=  66.4   Y(17)=       2.716
```

```
X(18)=  85.5   Y(18)=        -.077
X(19)=  15.2   Y(19)=        1.603
X(20)=   6.3   Y(20)=        1.599
T(A0)=        .098 T(A1)=     -.661
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=1) WITH NEGATIVE REGRESSION

```
A0=   1.0 EST A0=     .977 A1=  -2.0 EST A1=  -2.004
X( 1)=  27.3   Y( 1)=      -52.967
X( 2)=  66.4   Y( 2)=     -135.245
X( 3)=  22.7   Y( 3)=      -44.887
X( 4)=  14.4   Y( 4)=      -28.454
X( 5)=  30.0   Y( 5)=      -60.105
X( 6)=  73.3   Y( 6)=     -145.602
X( 7)=  99.2   Y( 7)=     -197.903
X( 8)=  11.0   Y( 8)=      -20.769
X( 9)=  50.3   Y( 9)=      -98.560
X(10)=  28.0   Y(10)=      -54.236
X(11)=  73.4   Y(11)=     -145.557
X(12)=  57.4   Y(12)=     -113.482
X(13)=  30.9   Y(13)=      -59.972
X(14)=  30.5   Y(14)=      -60.387
X(15)=  19.2   Y(15)=      -36.887
X(16)=  48.6   Y(16)=      -96.831
X(17)=  57.5   Y(17)=     -113.981
X(18)=  67.6   Y(18)=     -134.895
X(19)=   1.0   Y(19)=       -1.934
X(20)=  90.9   Y(20)=     -180.658
T(A0)=      -.051 T(A1)=      -.488

A0=   1.0 EST A0=     .719 A1=  -2.0 EST A1=  -1.994
X( 1)=  27.3   Y( 1)=      -53.627
X( 2)=  66.4   Y( 2)=     -133.377
X( 3)=  22.7   Y( 3)=      -43.891
X( 4)=  14.4   Y( 4)=      -26.730
X( 5)=  30.0   Y( 5)=      -59.054
X( 6)=  73.3   Y( 6)=     -144.622
X( 7)=  99.2   Y( 7)=     -196.395
X( 8)=  11.0   Y( 8)=      -22.134
X( 9)=  50.3   Y( 9)=      -99.197
X(10)=  28.0   Y(10)=      -56.544
X(11)=  73.4   Y(11)=     -146.937
X(12)=  57.4   Y(12)=     -113.334
X(13)=  30.9   Y(13)=      -62.696
X(14)=  30.5   Y(14)=      -59.383
X(15)=  19.2   Y(15)=      -36.555
X(16)=  48.6   Y(16)=      -96.572
X(17)=  57.5   Y(17)=     -113.193
X(18)=  67.6   Y(18)=     -133.979
```

```
X(19)=   1.0   Y(19)=      -1.290
X(20)=  90.9   Y(20)=    -180.686
T(A0)=      -.673 T(A1)=     .643

A0=   1.0 EST A0=   1.573 A1=  -2.0 EST A1=  -2.004
X( 1)=  27.3   Y( 1)=     -52.974
X( 2)=  66.4   Y( 2)=    -131.581
X( 3)=  22.7   Y( 3)=     -42.873
X( 4)=  14.4   Y( 4)=     -26.409
X( 5)=  30.0   Y( 5)=     -59.350
X( 6)=  73.3   Y( 6)=    -144.457
X( 7)=  99.2   Y( 7)=    -197.688
X( 8)=  11.0   Y( 8)=     -19.803
X( 9)=  50.3   Y( 9)=     -97.164
X(10)=  28.0   Y(10)=     -55.729
X(11)=  73.4   Y(11)=    -145.259
X(12)=  57.4   Y(12)=    -114.714
X(13)=  30.9   Y(13)=     -60.054
X(14)=  30.5   Y(14)=     -60.639
X(15)=  19.2   Y(15)=     -36.628
X(16)=  48.6   Y(16)=     -97.382
X(17)=  57.5   Y(17)=    -113.661
X(18)=  67.6   Y(18)=    -135.025
X(19)=   1.0   Y(19)=       -.833
X(20)=  90.9   Y(20)=    -179.647
T(A0)=      1.338 T(A1)=    -.563

A0=   1.0 EST A0=    .505 A1=  -2.0 EST A1=  -1.994
X( 1)=  27.3   Y( 1)=     -53.945
X( 2)=  66.4   Y( 2)=    -131.086
X( 3)=  22.7   Y( 3)=     -44.393
X( 4)=  14.4   Y( 4)=     -29.424
X( 5)=  30.0   Y( 5)=     -61.339
X( 6)=  73.3   Y( 6)=    -146.900
X( 7)=  99.2   Y( 7)=    -196.065
X( 8)=  11.0   Y( 8)=     -19.595
X( 9)=  50.3   Y( 9)=     -99.850
Y(10)=  28.0   Y(10)=     -55.790
X(11)=  73.4   Y(11)=    -145.175
X(12)=  57.4   Y(12)=    -112.964
X(13)=  30.9   Y(13)=     -63.118
X(14)=  30.5   Y(14)=     -60.997
X(15)=  19.2   Y(15)=     -34.761
X(16)=  48.6   Y(16)=     -95.769
X(17)=  57.5   Y(17)=    -113.783
X(18)=  67.6   Y(18)=    -135.361
X(19)=   1.0   Y(19)=      -1.864
X(20)=  90.9   Y(20)=    -181.851
T(A0)=      -.861 T(A1)=     .511

A0=   1.0 EST A0=   1.256 A1=  -2.0 EST A1=  -1.995
X( 1)=  27.3   Y( 1)=     -51.484
X( 2)=  66.4   Y( 2)=    -131.321
```

```
X( 3)=  22.7   Y( 3)=      -44.523
X( 4)=  14.4   Y( 4)=      -28.650
X( 5)=  30.0   Y( 5)=      -58.862
X( 6)=  73.3   Y( 6)=     -145.919
X( 7)=  99.2   Y( 7)=     -196.780
X( 8)=  11.0   Y( 8)=      -21.206
X( 9)=  50.3   Y( 9)=      -99.557
X(10)=  28.0   Y(10)=      -53.793
X(11)=  73.4   Y(11)=     -144.673
X(12)=  57.4   Y(12)=     -114.159
X(13)=  30.9   Y(13)=      -61.209
X(14)=  30.5   Y(14)=      -59.184
X(15)=  19.2   Y(15)=      -36.244
X(16)=  48.6   Y(16)=      -95.748
X(17)=  57.5   Y(17)=     -113.458
X(18)=  67.6   Y(18)=     -132.932
X(19)=   1.0   Y(19)=        -.531
X(20)=  90.9   Y(20)=     -179.814
T(A0)=        .794 T(A1)=     .724
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=2) WITH POSITIVE REGRESSION

```
A0=   2.0 EST A0=   1.322 A1=   2.0 EST A1=   2.013
X( 1)=  95.0   Y( 1)=      193.313
X( 2)=  68.8   Y( 2)=      140.246
X( 3)=  86.2   Y( 3)=      174.050
X( 4)=  83.5   Y( 4)=      167.004
X( 5)=  59.1   Y( 5)=      119.588
X( 6)=  11.4   Y( 6)=       26.283
X( 7)=  53.0   Y( 7)=      109.969
X( 8)=  33.0   Y( 8)=       66.524
X( 9)=  20.8   Y( 9)=       42.430
X(10)=   4.2   Y(10)=        8.967
X(11)=  83.9   Y(11)=      175.214
X(12)=  93.6   Y(12)=      188.251
X(13)=  31.8   Y(13)=       64.759
X(14)=  55.3   Y(14)=      116.017
X(15)=  43.3   Y(15)=       88.106
X(16)=  46.4   Y(16)=       95.904
X(17)=  86.5   Y(17)=      174.894
X(18)=  54.4   Y(18)=      106.353
X(19)=  63.4   Y(19)=      130.564
X(20)=  72.3   Y(20)=      144.804
T(A0)=       -.582 T(A1)=     .710

A0=   2.0 EST A0=   1.728 A1=   2.0 EST A1=   2.008
X( 1)=  95.0   Y( 1)=      190.555
X( 2)=  68.8   Y( 2)=      140.096
X( 3)=  86.2   Y( 3)=      176.108
```

```
X( 4)=  83.5   Y( 4)=      170.870
X( 5)=  59.1   Y( 5)=      118.863
X( 6)=  11.4   Y( 6)=       26.566
X( 7)=  53.0   Y( 7)=      108.859
X( 8)=  33.0   Y( 8)=       67.623
X( 9)=  20.8   Y( 9)=       43.337
X(10)=   4.2   Y(10)=        7.281
X(11)=  83.9   Y(11)=      166.536
X(12)=  93.6   Y(12)=      190.181
X(13)=  31.8   Y(13)=       64.897
X(14)=  55.3   Y(14)=      113.963
X(15)=  43.3   Y(15)=       87.460
X(16)=  46.4   Y(16)=       96.267
X(17)=  86.5   Y(17)=      173.864
X(18)=  54.4   Y(18)=      111.532
X(19)=  63.4   Y(19)=      133.550
X(20)=  72.3   Y(20)=      147.198
T(A0)=      -.268 T(A1)=      .504

A0=   2.0 EST A0=    3.242 A1=   2.0 EST A1=   1.983
X( 1)=  95.0   Y( 1)=      193.957
X( 2)=  68.8   Y( 2)=      142.107
X( 3)=  86.2   Y( 3)=      174.326
X( 4)=  83.5   Y( 4)=      170.896
X( 5)=  59.1   Y( 5)=      122.297
X( 6)=  11.4   Y( 6)=       25.008
X( 7)=  53.0   Y( 7)=      105.909
X( 8)=  33.0   Y( 8)=       70.881
X( 9)=  20.8   Y( 9)=       42.403
X(10)=   4.2   Y(10)=        9.755
X(11)=  83.9   Y(11)=      170.018
X(12)=  93.6   Y(12)=      186.272
X(13)=  31.8   Y(13)=       69.195
X(14)=  55.3   Y(14)=      112.069
X(15)=  43.3   Y(15)=       88.974
X(16)=  46.4   Y(16)=       98.789
X(17)=  86.5   Y(17)=      170.994
X(18)=  54.4   Y(18)=      110.870
X(19)=  63.4   Y(19)=      128.696
X(20)=  72.3   Y(20)=      143.753
T(A0)=      1.048 T(A1)=     -.905

A0=   2.0 EST A0=     .981 A1=   2.0 EST A1=   2.017
X( 1)=  95.0   Y( 1)=      189.519
X( 2)=  68.8   Y( 2)=      142.277
X( 3)=  86.2   Y( 3)=      176.704
X( 4)=  83.5   Y( 4)=      167.083
X( 5)=  59.1   Y( 5)=      119.891
X( 6)=  11.4   Y( 6)=       25.610
X( 7)=  53.0   Y( 7)=      109.119
X( 8)=  33.0   Y( 8)=       68.299
X( 9)=  20.8   Y( 9)=       39.629
X(10)=   4.2   Y(10)=        8.390
```

```
X(11)=  83.9   Y(11)=      169.661
X(12)=  93.6   Y(12)=      190.522
X(13)=  31.8   Y(13)=       63.654
X(14)=  55.3   Y(14)=      110.336
X(15)=  43.3   Y(15)=       88.648
X(16)=  46.4   Y(16)=       97.471
X(17)=  86.5   Y(17)=      174.284
X(18)=  54.4   Y(18)=      112.368
X(19)=  63.4   Y(19)=      128.002
X(20)=  72.3   Y(20)=      150.467
T(A0)=      -.930 T(A1)=     1.032

A0=   2.0 EST A0=     .655 A1=   2.0 EST A1=   2.011
X( 1)=  95.0   Y( 1)=      195.242
X( 2)=  68.8   Y( 2)=      136.607
X( 3)=  86.2   Y( 3)=      175.243
X( 4)=  83.5   Y( 4)=      169.429
X( 5)=  59.1   Y( 5)=      119.645
X( 6)=  11.4   Y( 6)=       24.372
X( 7)=  53.0   Y( 7)=      108.490
X( 8)=  33.0   Y( 8)=       65.877
X( 9)=  20.8   Y( 9)=       41.015
X(10)=   4.2   Y(10)=        9.184
X(11)=  83.9   Y(11)=      165.463
X(12)=  93.6   Y(12)=      190.932
X(13)=  31.8   Y(13)=       66.272
X(14)=  55.3   Y(14)=      108.762
X(15)=  43.3   Y(15)=       90.482
X(16)=  46.4   Y(16)=       94.313
X(17)=  86.5   Y(17)=      171.735
X(18)=  54.4   Y(18)=      110.026
X(19)=  63.4   Y(19)=      129.469
X(20)=  72.3   Y(20)=      145.341
T(A0)=     -1.225 T(A1)=        .652
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=2) WITH HORIZONTAL REGRESSION

```
A0=   2.0 EST A0=   3.336 A1=   0.0 EST A1=   -.025
X( 1)=  66.6   Y( 1)=         -.875
X( 2)=  19.2   Y( 2)=         3.497
X( 3)=  44.4   Y( 3)=         -.458
X( 4)=  64.2   Y( 4)=         4.935
X( 5)=  70.5   Y( 5)=         3.360
X( 6)=  84.0   Y( 6)=          .495
X( 7)=  46.7   Y( 7)=         2.020
X( 8)=  26.7   Y( 8)=         7.616
X( 9)=  93.7   Y( 9)=        -1.037
X(10)=  49.5   Y(10)=         3.738
X(11)=  19.5   Y(11)=         3.625
```

```
X(12)=  46.6   Y(12)=        .302
X(13)=  74.3   Y(13)=       -.549
X(14)=   7.1   Y(14)=        .748
X(15)=  33.3   Y(15)=       3.877
X(16)=   4.2   Y(16)=       2.516
X(17)=  66.4   Y(17)=       2.345
X(18)=  85.5   Y(18)=       3.045
X(19)=  15.2   Y(19)=        .295
X(20)=   6.3   Y(20)=       3.775
T(A0)=       1.406 T(A1)=  -1.442

A0=   2.0 EST A0=   2.295 A1=   0.0 EST A1=   -.015
X( 1)=  66.6   Y( 1)=       1.166
X( 2)=  19.2   Y( 2)=        .147
X( 3)=  44.4   Y( 3)=        .399
X( 4)=  64.2   Y( 4)=      -2.398
X( 5)=  70.5   Y( 5)=       -.565
X( 6)=  84.0   Y( 6)=       -.422
X( 7)=  46.7   Y( 7)=      -2.289
X( 8)=  26.7   Y( 8)=       7.514
X( 9)=  93.7   Y( 9)=       2.668
X(10)=  49.5   Y(10)=        .852
X(11)=  19.5   Y(11)=       3.747
X(12)=  46.6   Y(12)=       1.033
X(13)=  74.3   Y(13)=       4.388
X(14)=   7.1   Y(14)=       3.494
X(15)=  33.3   Y(15)=       2.031
X(16)=   4.2   Y(16)=       1.678
X(17)=  66.4   Y(17)=       2.115
X(18)=  85.5   Y(18)=       3.023
X(19)=  15.2   Y(19)=        .081
X(20)=   6.3   Y(20)=       2.970
T(A0)=        .290 T(A1)=   -.819

A0=   2.0 EST A0=   3.373 A1=   0.0 EST A1=   -.024
X( 1)=  66.6   Y( 1)=       -.631
X( 2)=  19.2   Y( 2)=        .958
X( 3)=  44.4   Y( 3)=       3.417
X( 4)=  64.2   Y( 4)=       2.428
X( 5)=  70.5   Y( 5)=       1.668
X( 6)=  84.0   Y( 6)=        .819
X( 7)=  46.7   Y( 7)=       -.439
X( 8)=  26.7   Y( 8)=       1.572
X( 9)=  93.7   Y( 9)=        .534
X(10)=  49.5   Y(10)=       4.127
X(11)=  19.5   Y(11)=       4.030
X(12)=  46.6   Y(12)=       1.923
X(13)=  74.3   Y(13)=       1.487
X(14)=   7.1   Y(14)=       4.401
X(15)=  33.3   Y(15)=        .345
X(16)=   4.2   Y(16)=       1.000
X(17)=  66.4   Y(17)=       4.045
X(18)=  85.5   Y(18)=       3.161
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```
X(19)=  15.2   Y(19)=        4.027
X(20)=   6.3   Y(20)=        6.324
T(A0)=      1.796 T(A1)=    -1.702

A0=   2.0 EST A0=   2.824 A1=   0.0 EST A1=   -.020
X( 1)=  66.6   Y( 1)=        1.731
X( 2)=  19.2   Y( 2)=        3.928
X( 3)=  44.4   Y( 3)=         .596
X( 4)=  64.2   Y( 4)=       -2.585
X( 5)=  70.5   Y( 5)=        2.062
X( 6)=  84.0   Y( 6)=        2.221
X( 7)=  46.7   Y( 7)=        3.571
X( 8)=  26.7   Y( 8)=        3.790
X( 9)=  93.7   Y( 9)=         .560
X(10)=  49.5   Y(10)=        3.561
X(11)=  19.5   Y(11)=        4.472
X(12)=  46.6   Y(12)=         .973
X(13)=  74.3   Y(13)=         .745
X(14)=   7.1   Y(14)=        -.532
X(15)=  33.3   Y(15)=        2.819
X(16)=   4.2   Y(16)=        2.482
X(17)=  66.4   Y(17)=        4.136
X(18)=  85.5   Y(18)=        -.540
X(19)=  15.2   Y(19)=        2.133
X(20)=   6.3   Y(20)=        1.838
T(A0)=      1.066 T(A1)=    -1.399

A0=   2.0 EST A0=   2.565 A1=   0.0 EST A1=   -.001
X( 1)=  66.6   Y( 1)=        4.253
X( 2)=  19.2   Y( 2)=        3.058
X( 3)=  44.4   Y( 3)=        -.065
X( 4)=  64.2   Y( 4)=        2.560
X( 5)=  70.5   Y( 5)=        2.617
X( 6)=  84.0   Y( 6)=        -.216
X( 7)=  46.7   Y( 7)=        5.741
X( 8)=  26.7   Y( 8)=        6.168
X( 9)=  93.7   Y( 9)=        2.747
X(10)=  49.5   Y(10)=        3.155
X(11)=  19.5   Y(11)=        -.925
X(12)=  46.6   Y(12)=        4.183
X(13)=  74.3   Y(13)=       -1.836
X(14)=   7.1   Y(14)=        4.693
X(15)=  33.3   Y(15)=        3.454
X(16)=   4.2   Y(16)=         .125
X(17)=  66.4   Y(17)=         .386
X(18)=  85.5   Y(18)=        5.918
X(19)=  15.2   Y(19)=        2.400
X(20)=   6.3   Y(20)=        1.512
T(A0)=       .545 T(A1)=     -.078
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)

FOR NORMAL (VARIANCE=2) WITH NEGATIVE REGRESSION

```
AO=   2.0 EST AO=   2.071 A1=  -2.0 EST A1=  -2.000
X( 1)=  27.3   Y( 1)=    -49.664
X( 2)=  66.4   Y( 2)=   -130.451
X( 3)=  22.7   Y( 3)=    -43.967
X( 4)=  14.4   Y( 4)=    -26.933
X( 5)=  30.0   Y( 5)=    -56.668
X( 6)=  73.3   Y( 6)=   -145.093
X( 7)=  99.2   Y( 7)=   -196.328
X( 8)=  11.0   Y( 8)=    -21.292
X( 9)=  50.3   Y( 9)=    -99.506
X(10)=  28.0   Y(10)=    -51.090
X(11)=  73.4   Y(11)=   -142.963
X(12)=  57.4   Y(12)=   -115.245
X(13)=  30.9   Y(13)=    -58.058
X(14)=  30.5   Y(14)=    -62.920
X(15)=  19.2   Y(15)=    -36.151
X(16)=  48.6   Y(16)=    -97.272
X(17)=  57.5   Y(17)=   -114.203
X(18)=  67.6   Y(18)=   -132.663
X(19)=   1.0   Y(19)=       .826
X(20)=  90.9   Y(20)=   -178.453
T(AO)=        .091 T(A1)=      -.024

AO=   2.0 EST AO=   1.046 A1=  -2.0 EST A1=  -1.979
X( 1)=  27.3   Y( 1)=    -52.822
X( 2)=  66.4   Y( 2)=   -127.000
X( 3)=  22.7   Y( 3)=    -40.309
X( 4)=  14.4   Y( 4)=    -27.466
X( 5)=  30.0   Y( 5)=    -57.794
X( 6)=  73.3   Y( 6)=   -143.211
X( 7)=  99.2   Y( 7)=   -195.838
X( 8)=  11.0   Y( 8)=    -20.594
X( 9)=  50.3   Y( 9)=    -97.000
X(10)=  28.0   Y(10)=    -55.175
X(11)=  73.4   Y(11)=   -146.041
X(12)=  57.4   Y(12)=   -113.715
X(13)=  30.9   Y(13)=    -60.319
X(14)=  30.5   Y(14)=    -63.373
X(15)=  19.2   Y(15)=    -35.197
X(16)=  48.6   Y(16)=    -95.310
X(17)=  57.5   Y(17)=   -113.633
X(18)=  67.6   Y(18)=   -133.885
X(19)=   1.0   Y(19)=     -2.587
X(20)=  90.9   Y(20)=   -178.458
T(AO)=     -1.222 T(A1)=      1.383

AO=   2.0 EST AO=   1.438 A1=  -2.0 EST A1=  -2.001
X( 1)=  27.3   Y( 1)=    -51.819
X( 2)=  66.4   Y( 2)=   -131.390
X( 3)=  22.7   Y( 3)=    -46.490
```

```
X( 4)=  14.4   Y( 4)=     -27.840
X( 5)=  30.0   Y( 5)=     -58.760
X( 6)=  73.3   Y( 6)=    -143.169
X( 7)=  99.2   Y( 7)=    -199.188
X( 8)=  11.0   Y( 8)=     -19.736
X( 9)=  50.3   Y( 9)=    -100.334
X(10)=  28.0   Y(10)=     -59.101
X(11)=  73.4   Y(11)=    -146.958
X(12)=  57.4   Y(12)=    -110.025
X(13)=  30.9   Y(13)=     -62.421
X(14)=  30.5   Y(14)=     -59.667
X(15)=  19.2   Y(15)=     -36.083
X(16)=  48.6   Y(16)=     -93.188
X(17)=  57.5   Y(17)=    -110.902
X(18)=  67.6   Y(18)=    -132.947
X(19)=   1.0   Y(19)=        .158
X(20)=  90.9   Y(20)=    -182.304
T(A0)=      -.621 T(A1)=     -.111

A0=   2.0 EST A0=      .897 A1=  -2.0 EST A1=  -1.984
X( 1)=  27.3   Y( 1)=     -50.657
X( 2)=  66.4   Y( 2)=    -131.620
X( 3)=  22.7   Y( 3)=     -46.512
X( 4)=  14.4   Y( 4)=     -28.054
X( 5)=  30.0   Y( 5)=     -57.565
X( 6)=  73.3   Y( 6)=    -142.967
X( 7)=  99.2   Y( 7)=    -196.377
X( 8)=  11.0   Y( 8)=     -18.718
X( 9)=  50.3   Y( 9)=    -101.507
X(10)=  28.0   Y(10)=     -54.867
X(11)=  73.4   Y(11)=    -143.716
X(12)=  57.4   Y(12)=    -110.175
X(13)=  30.9   Y(13)=     -62.363
X(14)=  30.5   Y(14)=     -59.801
X(15)=  19.2   Y(15)=     -36.808
X(16)=  48.6   Y(16)=     -94.905
X(17)=  57.5   Y(17)=    -114.012
X(18)=  67.6   Y(18)=    -131.848
X(19)=   1.0   Y(19)=      -2.934
X(20)=  90.9   Y(20)=    -181.990
T(A0)=     -1.452 T(A1)=     1.061

A0=   2.0 EST A0=    1.955 A1=  -2.0 EST A1=  -2.008
X( 1)=  27.3   Y( 1)=     -51.335
X( 2)=  66.4   Y( 2)=    -137.690
X( 3)=  22.7   Y( 3)=     -44.374
X( 4)=  14.4   Y( 4)=     -28.108
X( 5)=  30.0   Y( 5)=     -60.211
X( 6)=  73.3   Y( 6)=    -144.604
X( 7)=  99.2   Y( 7)=    -197.407
X( 8)=  11.0   Y( 8)=     -19.539
X( 9)=  50.3   Y( 9)=     -96.521
X(10)=  28.0   Y(10)=     -52.473
```

```
X(11)=  73.4   Y(11)=    -144.314
X(12)=  57.4   Y(12)=    -112.164
X(13)=  30.9   Y(13)=     -58.145
X(14)=  30.5   Y(14)=     -59.775
X(15)=  19.2   Y(15)=     -35.374
X(16)=  48.6   Y(16)=     -96.463
X(17)=  57.5   Y(17)=    -112.962
X(18)=  67.6   Y(18)=    -134.590
X(19)=   1.0   Y(19)=      -1.868
X(20)=  90.9   Y(20)=    -179.516
T(A0)=      -.051 T(A1)=      -.488
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=4) WITH POSITIVE REGRESSION

```
A0=   4.0 EST A0=   3.026 A1=   2.0 EST A1=   2.001
X( 1)=  95.0   Y( 1)=     195.493
X( 2)=  68.8   Y( 2)=     140.000
X( 3)=  86.2   Y( 3)=     175.047
X( 4)=  83.5   Y( 4)=     168.595
X( 5)=  59.1   Y( 5)=     128.804
X( 6)=  11.4   Y( 6)=      27.834
X( 7)=  53.0   Y( 7)=     110.245
X( 8)=  33.0   Y( 8)=      69.596
X( 9)=  20.8   Y( 9)=      44.049
X(10)=   4.2   Y(10)=       8.562
X(11)=  83.9   Y(11)=     167.895
X(12)=  93.6   Y(12)=     188.810
X(13)=  31.8   Y(13)=      67.665
X(14)=  55.3   Y(14)=     109.422
X(15)=  43.3   Y(15)=      87.839
X(16)=  46.4   Y(16)=      95.477
X(17)=  86.5   Y(17)=     175.495
X(18)=  54.4   Y(18)=     112.855
X(19)=  63.4   Y(19)=     129.515
X(20)=  72.3   Y(20)=     150.436
T(A0)=      -.701 T(A1)=       .051

A0=   4.0 EST A0=   4.326 A1=   2.0 EST A1=   2.003
X( 1)=  95.0   Y( 1)=     194.778
X( 2)=  68.8   Y( 2)=     144.500
X( 3)=  86.2   Y( 3)=     171.964
X( 4)=  83.5   Y( 4)=     177.128
X( 5)=  59.1   Y( 5)=     120.153
X( 6)=  11.4   Y( 6)=      37.199
X( 7)=  53.0   Y( 7)=     104.825
X( 8)=  33.0   Y( 8)=      68.593
X( 9)=  20.8   Y( 9)=      46.661
X(10)=   4.2   Y(10)=       9.990
X(11)=  83.9   Y(11)=     175.340
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```
X(12)=  93.6   Y(12)=      193.471
X(13)=  31.8   Y(13)=       68.742
X(14)=  55.3   Y(14)=      114.114
X(15)=  43.3   Y(15)=       83.347
X(16)=  46.4   Y(16)=       97.001
X(17)=  86.5   Y(17)=      174.236
X(18)=  54.4   Y(18)=      112.011
X(19)=  63.4   Y(19)=      132.287
X(20)=  72.3   Y(20)=      156.024
T(A0)=       .140 T(A1)=      .095

A0=   4.0 EST A0=   1.745 A1=   2.0 EST A1=   2.037
X( 1)=  95.0   Y( 1)=      206.382
X( 2)=  68.8   Y( 2)=      133.321
X( 3)=  86.2   Y( 3)=      181.200
X( 4)=  83.5   Y( 4)=      165.980
X( 5)=  59.1   Y( 5)=      115.821
X( 6)=  11.4   Y( 6)=       22.883
X( 7)=  53.0   Y( 7)=      107.726
X( 8)=  33.0   Y( 8)=       67.909
X( 9)=  20.8   Y( 9)=       45.594
X(10)=   4.2   Y(10)=       19.739
X(11)=  83.9   Y(11)=      175.104
X(12)=  93.6   Y(12)=      198.451
X(13)=  31.8   Y(13)=       62.138
X(14)=  55.3   Y(14)=      111.127
X(15)=  43.3   Y(15)=       91.176
X(16)=  46.4   Y(16)=      106.846
X(17)=  86.5   Y(17)=      177.296
X(18)=  54.4   Y(18)=      111.488
X(19)=  63.4   Y(19)=      123.380
X(20)=  72.3   Y(20)=      145.933
T(A0)=      -.710 T(A1)=      .742

A0=   4.0 EST A0=   4.080 A1=   2.0 EST A1=   1.987
X( 1)=  95.0   Y( 1)=      190.307
X( 2)=  68.8   Y( 2)=      150.461
X( 3)=  86.2   Y( 3)=      174.757
X( 4)=  83.5   Y( 4)=      179.153
X( 5)=  59.1   Y( 5)=      127.810
X( 6)=  11.4   Y( 6)=       28.888
X( 7)=  53.0   Y( 7)=      114.946
X( 8)=  33.0   Y( 8)=       63.546
X( 9)=  20.8   Y( 9)=       40.846
X(10)=   4.2   Y(10)=       13.807
X(11)=  83.9   Y(11)=      163.189
X(12)=  93.6   Y(12)=      187.751
X(13)=  31.8   Y(13)=       67.855
X(14)=  55.3   Y(14)=      116.459
X(15)=  43.3   Y(15)=       91.324
X(16)=  46.4   Y(16)=       93.010
X(17)=  86.5   Y(17)=      176.677
X(18)=  54.4   Y(18)=      111.284
```

```
X(19)=  63.4   Y(19)=       122.792
X(20)=  72.3   Y(20)=       144.161
T(A0)=       .029 T(A1)=      -.291

A0=   4.0 EST A0=   5.334 A1=   2.0 EST A1=   1.969
X( 1)=  95.0   Y( 1)=       190.551
X( 2)=  68.8   Y( 2)=       143.922
X( 3)=  86.2   Y( 3)=       180.633
X( 4)=  83.5   Y( 4)=       164.645
X( 5)=  59.1   Y( 5)=       124.118
X( 6)=  11.4   Y( 6)=        31.212
X( 7)=  53.0   Y( 7)=       114.487
X( 8)=  33.0   Y( 8)=        71.502
X( 9)=  20.8   Y( 9)=        48.419
X(10)=   4.2   Y(10)=        12.196
X(11)=  83.9   Y(11)=       171.593
X(12)=  93.6   Y(12)=       189.372
X(13)=  31.8   Y(13)=        65.891
X(14)=  55.3   Y(14)=       110.112
X(15)=  43.3   Y(15)=        87.793
X(16)=  46.4   Y(16)=        95.494
X(17)=  86.5   Y(17)=       180.377
X(18)=  54.4   Y(18)=       107.401
X(19)=  63.4   Y(19)=       130.525
X(20)=  72.3   Y(20)=       142.710
T(A0)=       .711 T(A1)=    -1.044


INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=4) WITH HORIZONTAL REGRESSION

A0=   4.0 EST A0=   4.343 A1=   0.0 EST A1=   -.010
X( 1)=  66.6   Y( 1)=         1.116
X( 2)=  19.2   Y( 2)=         4.102
X( 3)=  44.4   Y( 3)=        -5.569
X( 4)=  64.2   Y( 4)=        11.458
X( 5)=  70.5   Y( 5)=        -1.452
X( 6)=  84.0   Y( 6)=         3.057
X( 7)=  46.7   Y( 7)=          .347
X( 8)=  26.7   Y( 8)=         1.779
X( 9)=  93.7   Y( 9)=         6.711
X(10)=  49.5   Y(10)=         2.504
X(11)=  19.5   Y(11)=         8.518
X(12)=  46.6   Y(12)=         8.112
X(13)=  74.3   Y(13)=         4.648
X(14)=   7.1   Y(14)=         9.484
X(15)=  33.3   Y(15)=         9.981
X(16)=   4.2   Y(16)=         1.499
X(17)=  66.4   Y(17)=         3.398
X(18)=  85.5   Y(18)=         3.037
X(19)=  15.2   Y(19)=         -.222
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X(20)=   6.3   Y(20)=        4.978
T(A0)=       .182 T(A1)=     -.290

A0=   4.0 EST A0=   2.807 A1=   0.0 EST A1=    .032
X( 1)=  66.6   Y( 1)=       -1.999
X( 2)=  19.2   Y( 2)=        2.203
X( 3)=  44.4   Y( 3)=        8.946
X( 4)=  64.2   Y( 4)=        9.590
X( 5)=  70.5   Y( 5)=        7.495
X( 6)=  84.0   Y( 6)=        6.021
X( 7)=  46.7   Y( 7)=        8.528
X( 8)=  26.7   Y( 8)=        6.375
X( 9)=  93.7   Y( 9)=       10.923
X(10)=  49.5   Y(10)=        1.532
X(11)=  19.5   Y(11)=        5.562
X(12)=  46.6   Y(12)=        2.373
X(13)=  74.3   Y(13)=        -.675
X(14)=   7.1   Y(14)=        3.777
X(15)=  33.3   Y(15)=        3.090
X(16)=   4.2   Y(16)=         .623
X(17)=  66.4   Y(17)=        3.738
X(18)=  85.5   Y(18)=        3.793
X(19)=  15.2   Y(19)=       -3.849
X(20)=   6.3   Y(20)=        8.167
T(A0)=      -.686 T(A1)=     1.011

A0=   4.0 EST A0=   5.876 A1=   0.0 EST A1=   -.021
X( 1)=  66.6   Y( 1)=        7.205
X( 2)=  19.2   Y( 2)=        4.623
X( 3)=  44.4   Y( 3)=        -.216
X( 4)=  64.2   Y( 4)=         .043
X( 5)=  70.5   Y( 5)=         .764
X( 6)=  84.0   Y( 6)=        1.306
X( 7)=  46.7   Y( 7)=        9.028
X( 8)=  26.7   Y( 8)=        7.292
X( 9)=  93.7   Y( 9)=        3.456
X(10)=  49.5   Y(10)=        8.881
X(11)=  19.5   Y(11)=       10.927
X(12)=  46.6   Y(12)=         .953
X(13)=  74.3   Y(13)=       10.321
X(14)=   7.1   Y(14)=        6.389
X(15)=  33.3   Y(15)=        4.518
X(16)=   4.2   Y(16)=         .068
X(17)=  66.4   Y(17)=        4.399
X(18)=  85.5   Y(18)=        4.870
X(19)=  15.2   Y(19)=        4.842
X(20)=   6.3   Y(20)=        7.675
T(A0)=      1.206 T(A1)=     -.757

A0=   4.0 EST A0=   2.683 A1=   0.0 EST A1=   -.011
X( 1)=  66.6   Y( 1)=        4.729
X( 2)=  19.2   Y( 2)=        -.635
X( 3)=  44.4   Y( 3)=        2.939
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X( 4)=   64.2   Y( 4)=        2.815
X( 5)=   70.5   Y( 5)=       -1.647
X( 6)=   84.0   Y( 6)=        4.910
X( 7)=   46.7   Y( 7)=       -2.150
X( 8)=   26.7   Y( 8)=         .528
X( 9)=   93.7   Y( 9)=        4.308
X(10)=   49.5   Y(10)=        4.549
X(11)=   19.5   Y(11)=         .611
X(12)=   46.6   Y(12)=        3.854
X(13)=   74.3   Y(13)=       -2.362
X(14)=    7.1   Y(14)=        1.322
X(15)=   33.3   Y(15)=        6.267
X(16)=    4.2   Y(16)=        3.832
X(17)=   66.4   Y(17)=       -2.620
X(18)=   85.5   Y(18)=        2.266
X(19)=   15.2   Y(19)=        1.855
X(20)=    6.3   Y(20)=        7.504
T(A0)=     -1.007 T(A1)=       -.482

A0=    4.0 EST A0=    5.516 A1=    0.0 EST A1=    -.038
X( 1)=   66.6   Y( 1)=       -1.425
X( 2)=   19.2   Y( 2)=       10.424
X( 3)=   44.4   Y( 3)=       -1.583
X( 4)=   64.2   Y( 4)=       -2.091
X( 5)=   70.5   Y( 5)=        8.261
X( 6)=   84.0   Y( 6)=         .835
X( 7)=   46.7   Y( 7)=        2.989
X( 8)=   26.7   Y( 8)=       10.085
X( 9)=   93.7   Y( 9)=        1.481
X(10)=   49.5   Y(10)=         .538
X(11)=   19.5   Y(11)=        2.616
X(12)=   46.6   Y(12)=        7.074
X(13)=   74.3   Y(13)=        9.274
X(14)=    7.1   Y(14)=         .574
X(15)=   33.3   Y(15)=         .335
X(16)=    4.2   Y(16)=        7.917
X(17)=   66.4   Y(17)=        6.679
X(18)=   85.5   Y(18)=        -.016
X(19)=   15.2   Y(19)=       11.187
X(20)=    6.3   Y(20)=        -.347
T(A0)=       .771 T(A1)=     -1.055
```

INDIVIDUAL SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=4) WITH NEGATIVE REGRESSION

```
A0=    4.0 EST A0=    3.833 A1=   -2.0 EST A1=   -1.981
X( 1)=   27.3   Y( 1)=      -49.861
X( 2)=   66.4   Y( 2)=     -134.994
X( 3)=   22.7   Y( 3)=      -39.187
X( 4)=   14.4   Y( 4)=      -23.479
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```
X( 5)=  30.0   Y( 5)=     -49.510
X( 6)=  73.3   Y( 6)=    -145.520
X( 7)=  99.2   Y( 7)=    -189.949
X( 8)=  11.0   Y( 8)=     -22.038
X( 9)=  50.3   Y( 9)=     -93.626
X(10)=  28.0   Y(10)=     -51.153
X(11)=  73.4   Y(11)=    -137.859
X(12)=  57.4   Y(12)=    -108.184
X(13)=  30.9   Y(13)=     -60.569
X(14)=  30.5   Y(14)=     -56.853
X(15)=  19.2   Y(15)=     -31.675
X(16)=  48.6   Y(16)=     -88.878
X(17)=  57.5   Y(17)=    -114.699
X(18)=  67.6   Y(18)=    -129.180
X(19)=   1.0   Y(19)=      -1.159
X(20)=  90.9   Y(20)=    -177.678
T(A0)=       -.108 T(A1)=      .626

A0=   4.0 EST A0=   2.509 A1=  -2.0 EST A1=  -1.951
X( 1)=  27.3   Y( 1)=     -47.377
X( 2)=  66.4   Y( 2)=    -127.294
X( 3)=  22.7   Y( 3)=     -39.070
X( 4)=  14.4   Y( 4)=     -19.746
X( 5)=  30.0   Y( 5)=     -54.961
X( 6)=  73.3   Y( 6)=    -144.955
X( 7)=  99.2   Y( 7)=    -184.169
X( 8)=  11.0   Y( 8)=     -15.841
X( 9)=  50.3   Y( 9)=     -91.813
X(10)=  28.0   Y(10)=     -58.524
X(11)=  73.4   Y(11)=    -143.214
X(12)=  57.4   Y(12)=    -112.324
X(13)=  30.9   Y(13)=     -60.292
X(14)=  30.5   Y(14)=     -60.960
X(15)=  19.2   Y(15)=     -36.967
X(16)=  48.6   Y(16)=     -92.153
X(17)=  57.5   Y(17)=    -104.759
X(18)=  67.6   Y(18)=    -126.823
X(19)=   1.0   Y(19)=      -3.187
X(20)=  90.9   Y(20)=    -180.890
T(A0)=       -.825 T(A1)=     1.408

A0=   4.0 EST A0=   3.790 A1=  -2.0 EST A1=  -2.012
X( 1)=  27.3   Y( 1)=     -56.572
X( 2)=  66.4   Y( 2)=    -127.273
X( 3)=  22.7   Y( 3)=     -38.634
X( 4)=  14.4   Y( 4)=     -23.694
X( 5)=  30.0   Y( 5)=     -60.093
X( 6)=  73.3   Y( 6)=    -144.071
X( 7)=  99.2   Y( 7)=    -194.068
X( 8)=  11.0   Y( 8)=     -13.325
X( 9)=  50.3   Y( 9)=     -89.681
X(10)=  28.0   Y(10)=     -53.576
X(11)=  73.4   Y(11)=    -144.250
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```
X(12)=  57.4   Y(12)=    -112.143
X(13)=  30.9   Y(13)=     -59.696
X(14)=  30.5   Y(14)=     -60.748
X(15)=  19.2   Y(15)=     -33.939
X(16)=  48.6   Y(16)=     -95.109
X(17)=  57.5   Y(17)=    -110.498
X(18)=  67.6   Y(18)=    -132.147
X(19)=   1.0   Y(19)=       -.894
X(20)=  90.9   Y(20)=    -183.781
T(A0)=      -.145 T(A1)=     -.437

A0=   4.0 EST A0=   3.691 A1=  -2.0 EST A1=  -1.981
X( 1)=  27.3   Y( 1)=     -57.448
X( 2)=  66.4   Y( 2)=    -122.933
X( 3)=  22.7   Y( 3)=     -37.878
X( 4)=  14.4   Y( 4)=     -23.321
X( 5)=  30.0   Y( 5)=     -56.904
X( 6)=  73.3   Y( 6)=    -146.866
X( 7)=  99.2   Y( 7)=    -195.648
X( 8)=  11.0   Y( 8)=     -14.488
X( 9)=  50.3   Y( 9)=    -103.228
X(10)=  28.0   Y(10)=     -48.307
X(11)=  73.4   Y(11)=    -140.965
X(12)=  57.4   Y(12)=    -107.643
X(13)=  30.9   Y(13)=     -62.779
X(14)=  30.5   Y(14)=     -56.215
X(15)=  19.2   Y(15)=     -30.590
X(16)=  48.6   Y(16)=     -89.744
X(17)=  57.5   Y(17)=    -111.918
X(18)=  67.6   Y(18)=    -125.150
X(19)=   1.0   Y(19)=      -2.282
X(20)=  90.9   Y(20)=    -174.353
T(A0)=      -.170 T(A1)=       .537

A0=   4.0 EST A0=   5.763 A1=  -2.0 EST A1=  -2.023
X( 1)=  27.3   Y( 1)=     -50.003
X( 2)=  66.4   Y( 2)=    -122.272
X( 3)=  22.7   Y( 3)=     -40.801
X( 4)=  14.4   Y( 4)=     -22.629
X( 5)=  30.0   Y( 5)=     -53.396
X( 6)=  73.3   Y( 6)=    -145.342
X( 7)=  99.2   Y( 7)=    -196.907
X( 8)=  11.0   Y( 8)=      -7.332
X( 9)=  50.3   Y( 9)=     -96.456
X(10)=  28.0   Y(10)=     -54.719
X(11)=  73.4   Y(11)=    -145.361
X(12)=  57.4   Y(12)=    -110.822
X(13)=  30.9   Y(13)=     -57.543
X(14)=  30.5   Y(14)=     -59.363
X(15)=  19.2   Y(15)=     -34.922
X(16)=  48.6   Y(16)=     -92.060
X(17)=  57.5   Y(17)=    -113.017
X(18)=  67.6   Y(18)=    -125.834
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```
X(19)=   1.0   Y(19)=         .650
X(20)=  90.9   Y(20)=     -176.604
T(A0)=      1.151 T(A1)=      -.791
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR EXPONENTIAL WITH POSITIVE REGRESSION

A0= 1.0 EST A0= 1.101 A1= 2.0 EST A1= 1.999

X(1)=	95.0	Y(1)=	191.171
X(2)=	68.8	Y(2)=	138.350
X(3)=	86.2	Y(3)=	172.738
X(4)=	83.5	Y(4)=	167.738
X(5)=	59.1	Y(5)=	118.981
X(6)=	11.4	Y(6)=	22.986
X(7)=	53.0	Y(7)=	107.200
X(8)=	33.0	Y(8)=	66.590
X(9)=	20.8	Y(9)=	41.626
X(10)=	4.2	Y(10)=	8.637
X(11)=	83.9	Y(11)=	168.596
X(12)=	93.6	Y(12)=	188.331
X(13)=	31.8	Y(13)=	64.879
X(14)=	55.3	Y(14)=	110.820
X(15)=	43.3	Y(15)=	88.304
X(16)=	46.4	Y(16)=	94.194
X(17)=	86.5	Y(17)=	173.092
X(18)=	54.4	Y(18)=	109.522
X(19)=	63.4	Y(19)=	127.734
X(20)=	72.3	Y(20)=	145.051
X(21)=	95.0	Y(21)=	190.622
X(22)=	68.8	Y(22)=	138.088
X(23)=	86.2	Y(23)=	172.547
X(24)=	83.5	Y(24)=	167.455
X(25)=	59.1	Y(25)=	118.270
X(26)=	11.4	Y(26)=	24.924
X(27)=	53.0	Y(27)=	107.378
X(28)=	33.0	Y(28)=	67.487
X(29)=	20.8	Y(29)=	41.601
X(30)=	4.2	Y(30)=	10.717
X(31)=	83.9	Y(31)=	170.117
X(32)=	93.6	Y(32)=	188.637
X(33)=	31.8	Y(33)=	63.876
X(34)=	55.3	Y(34)=	110.817
X(35)=	43.3	Y(35)=	86.795
X(36)=	46.4	Y(36)=	93.894
X(37)=	86.5	Y(37)=	174.043
X(38)=	54.4	Y(38)=	109.226
X(39)=	63.4	Y(39)=	127.126
X(40)=	72.3	Y(40)=	148.372
X(41)=	95.0	Y(41)=	193.227
X(42)=	68.8	Y(42)=	139.128
X(43)=	86.2	Y(43)=	172.423
X(44)=	83.5	Y(44)=	167.433

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X( 45)=  59.1   Y( 45)=   119.797
X( 46)=  11.4   Y( 46)=    24.263
X( 47)=  53.0   Y( 47)=   106.436
X( 48)=  33.0   Y( 48)=    66.928
X( 49)=  20.8   Y( 49)=    42.676
X( 50)=   4.2   Y( 50)=     8.925
X( 51)=  83.9   Y( 51)=   168.201
X( 52)=  93.6   Y( 52)=   188.436
X( 53)=  31.8   Y( 53)=    66.175
X( 54)=  55.3   Y( 54)=   111.650
X( 55)=  43.3   Y( 55)=    86.744
X( 56)=  46.4   Y( 56)=    94.414
X( 57)=  86.5   Y( 57)=   174.905
X( 58)=  54.4   Y( 58)=   109.125
X( 59)=  63.4   Y( 59)=   126.868
X( 60)=  72.3   Y( 60)=   144.790
X( 61)=  95.0   Y( 61)=   190.868
X( 62)=  68.8   Y( 62)=   140.091
X( 63)=  86.2   Y( 63)=   173.191
X( 64)=  83.5   Y( 64)=   167.329
X( 65)=  59.1   Y( 65)=   118.339
X( 66)=  11.4   Y( 66)=    22.834
X( 67)=  53.0   Y( 67)=   106.123
X( 68)=  33.0   Y( 68)=    67.339
X( 69)=  20.8   Y( 69)=    42.111
X( 70)=   4.2   Y( 70)=    11.120
X( 71)=  83.9   Y( 71)=   168.373
X( 72)=  93.6   Y( 72)=   187.584
X( 73)=  31.8   Y( 73)=    63.785
X( 74)=  55.3   Y( 74)=   112.039
X( 75)=  43.3   Y( 75)=    86.939
X( 76)=  46.4   Y( 76)=    95.995
X( 77)=  86.5   Y( 77)=   173.102
X( 78)=  54.4   Y( 78)=   109.505
X( 79)=  63.4   Y( 79)=   127.646
X( 80)=  72.3   Y( 80)=   145.993
X( 81)=  95.0   Y( 81)=   191.284
X( 82)=  68.8   Y( 82)=   142.099
X( 83)=  86.2   Y( 83)=   172.766
X( 84)=  83.5   Y( 84)=   170.031
X( 85)=  59.1   Y( 85)=   118.827
X( 86)=  11.4   Y( 86)=    24.898
X( 87)=  53.0   Y( 87)=   107.010
X( 88)=  33.0   Y( 88)=    69.629
X( 89)=  20.8   Y( 89)=    42.579
X( 90)=   4.2   Y( 90)=     9.529
X( 91)=  83.9   Y( 91)=   169.511
X( 92)=  93.6   Y( 92)=   187.696
X( 93)=  31.8   Y( 93)=    64.075
X( 94)=  55.3   Y( 94)=   111.921
X( 95)=  43.3   Y( 95)=    86.871
X( 96)=  46.4   Y( 96)=    95.605
X( 97)=  86.5   Y( 97)=   174.547
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```
X( 98)=   54.4    Y( 98)=      109.063
X( 99)=   63.4    Y( 99)=      127.298
X(100)=   72.3    Y(100)=      145.318
T(AO)=        .451 T(A1)=      -.352
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR EXPONENTIAL WITH HORIZONTAL REGRESSION

```
A0=    1.0 EST A0=     .966 A1=    0.0 EST A1=       .001
X(  1)=   66.6    Y(  1)=        1.558
X(  2)=   19.2    Y(  2)=         .220
X(  3)=   44.4    Y(  3)=        2.339
X(  4)=   64.2    Y(  4)=         .180
X(  5)=   70.5    Y(  5)=         .229
X(  6)=   84.0    Y(  6)=         .689
X(  7)=   46.7    Y(  7)=         .717
X(  8)=   26.7    Y(  8)=         .118
X(  9)=   93.7    Y(  9)=        1.317
X( 10)=   49.5    Y( 10)=        1.815
X( 11)=   19.5    Y( 11)=         .082
X( 12)=   46.6    Y( 12)=        1.293
X( 13)=   74.3    Y( 13)=        3.014
X( 14)=    7.1    Y( 14)=        1.431
X( 15)=   33.3    Y( 15)=         .879
X( 16)=    4.2    Y( 16)=         .781
X( 17)=   66.4    Y( 17)=         .385
X( 18)=   85.5    Y( 18)=        1.064
X( 19)=   15.2    Y( 19)=         .140
X( 20)=    6.3    Y( 20)=         .295
X( 21)=   66.6    Y( 21)=         .197
X( 22)=   19.2    Y( 22)=        1.366
X( 23)=   44.4    Y( 23)=        2.769
X( 24)=   64.2    Y( 24)=        1.276
X( 25)=   70.5    Y( 25)=        1.374
X( 26)=   84.0    Y( 26)=         .101
X( 27)=   46.7    Y( 27)=         .423
X( 28)=   26.7    Y( 28)=        3.030
X( 29)=   93.7    Y( 29)=         .131
X( 30)=   49.5    Y( 30)=         .954
X( 31)=   19.5    Y( 31)=        1.690
X( 32)=   46.6    Y( 32)=         .130
X( 33)=   74.3    Y( 33)=         .337
X( 34)=    7.1    Y( 34)=         .047
X( 35)=   33.3    Y( 35)=        2.642
X( 36)=    4.2    Y( 36)=         .838
X( 37)=   66.4    Y( 37)=         .101
X( 38)=   85.5    Y( 38)=        3.731
X( 39)=   15.2    Y( 39)=         .487
X( 40)=    6.3    Y( 40)=        1.522
X( 41)=   66.6    Y( 41)=        2.379
X( 42)=   19.2    Y( 42)=        3.279
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X( 43)=  44.4   Y( 43)=        .323
X( 44)=  64.2   Y( 44)=       1.532
X( 45)=  70.5   Y( 45)=        .426
X( 46)=  84.0   Y( 46)=        .389
X( 47)=  46.7   Y( 47)=        .328
X( 48)=  26.7   Y( 48)=        .172
X( 49)=  93.7   Y( 49)=        .095
X( 50)=  49.5   Y( 50)=       1.317
X( 51)=  19.5   Y( 51)=       1.234
X( 52)=  46.6   Y( 52)=        .194
X( 53)=  74.3   Y( 53)=       3.191
X( 54)=   7.1   Y( 54)=       1.862
X( 55)=  33.3   Y( 55)=        .202
X( 56)=   4.2   Y( 56)=        .110
X( 57)=  66.4   Y( 57)=       2.231
X( 58)=  85.5   Y( 58)=        .644
X( 59)=  15.2   Y( 59)=        .224
X( 60)=   6.3   Y( 60)=        .048
X( 61)=  66.6   Y( 61)=        .750
X( 62)=  19.2   Y( 62)=       1.054
X( 63)=  44.4   Y( 63)=       1.255
X( 64)=  64.2   Y( 64)=        .061
X( 65)=  70.5   Y( 65)=       1.837
X( 66)=  84.0   Y( 66)=        .026
X( 67)=  46.7   Y( 67)=        .330
X( 68)=  26.7   Y( 68)=       2.806
X( 69)=  93.7   Y( 69)=        .437
X( 70)=  49.5   Y( 70)=       1.736
X( 71)=  19.5   Y( 71)=        .126
X( 72)=  46.6   Y( 72)=        .361
X( 73)=  74.3   Y( 73)=       1.875
X( 74)=   7.1   Y( 74)=        .210
X( 75)=  33.3   Y( 75)=        .006
X( 76)=   4.2   Y( 76)=        .089
X( 77)=  66.4   Y( 77)=       3.811
X( 78)=  85.5   Y( 78)=        .117
X( 79)=  15.2   Y( 79)=       1.874
X( 80)=   6.3   Y( 80)=        .479
X( 81)=  66.6   Y( 81)=       2.796
X( 82)=  19.2   Y( 82)=        .210
X( 83)=  44.4   Y( 83)=        .850
X( 84)=  64.2   Y( 84)=        .077
X( 85)=  70.5   Y( 85)=       3.551
X( 86)=  84.0   Y( 86)=        .737
X( 87)=  46.7   Y( 87)=       2.743
X( 88)=  26.7   Y( 88)=       1.678
X( 89)=  93.7   Y( 89)=        .003
X( 90)=  49.5   Y( 90)=        .030
X( 91)=  19.5   Y( 91)=       1.761
X( 92)=  46.6   Y( 92)=        .631
X( 93)=  74.3   Y( 93)=        .296
X( 94)=   7.1   Y( 94)=        .845
X( 95)=  33.3   Y( 95)=        .634
```

```
X( 96)=   4.2   Y( 96)=        .110
X( 97)=  66.4   Y( 97)=        .073
X( 98)=  85.5   Y( 98)=        .170
X( 99)=  15.2   Y( 99)=       3.201
X(100)=   6.3   Y(100)=        .988
T(A0)=      -.170 T(A1)=     .345
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR EXPONENTIAL WITH NEGATIVE REGRESSION

```
A0=    1.0 EST A0=    1.243 A1=  -2.0 EST A1=  -2.004
X(  1)=  27.3   Y(  1)=     -51.124
X(  2)=  66.4   Y(  2)=    -132.430
X(  3)=  22.7   Y(  3)=     -44.820
X(  4)=  14.4   Y(  4)=     -28.340
X(  5)=  30.0   Y(  5)=     -57.053
X(  6)=  73.3   Y(  6)=    -146.290
X(  7)=  99.2   Y(  7)=    -198.118
X(  8)=  11.0   Y(  8)=     -21.588
X(  9)=  50.3   Y(  9)=     -99.870
X( 10)=  28.0   Y( 10)=     -53.668
X( 11)=  73.4   Y( 11)=    -146.046
X( 12)=  57.4   Y( 12)=    -114.393
X( 13)=  30.9   Y( 13)=     -61.498
X( 14)=  30.5   Y( 14)=     -59.555
X( 15)=  19.2   Y( 15)=     -38.082
X( 16)=  48.6   Y( 16)=     -94.965
X( 17)=  57.5   Y( 17)=    -113.999
X( 18)=  67.6   Y( 18)=    -134.930
X( 19)=   1.0   Y( 19)=      -1.145
X( 20)=  90.9   Y( 20)=    -180.565
X( 21)=  27.3   Y( 21)=     -50.929
X( 22)=  66.4   Y( 22)=    -132.015
X( 23)=  22.7   Y( 23)=     -44.327
X( 24)=  14.4   Y( 24)=     -27.788
X( 25)=  30.0   Y( 25)=     -59.295
X( 26)=  73.3   Y( 26)=    -145.203
X( 27)=  99.2   Y( 27)=    -196.415
X( 28)=  11.0   Y( 28)=     -20.830
X( 29)=  50.3   Y( 29)=    -100.207
X( 30)=  28.0   Y( 30)=     -54.535
X( 31)=  73.4   Y( 31)=    -145.751
X( 32)=  57.4   Y( 32)=    -114.756
X( 33)=  30.9   Y( 33)=     -61.270
X( 34)=  30.5   Y( 34)=     -60.588
X( 35)=  19.2   Y( 35)=     -37.484
X( 36)=  48.6   Y( 36)=     -97.169
X( 37)=  57.5   Y( 37)=    -114.020
X( 38)=  67.6   Y( 38)=    -135.154
X( 39)=   1.0   Y( 39)=      -1.466
X( 40)=  90.9   Y( 40)=    -181.641
```

X(41)=	27.3	Y(41)=	-53.385
X(42)=	66.4	Y(42)=	-132.420
X(43)=	22.7	Y(43)=	-42.966
X(44)=	14.4	Y(44)=	-28.752
X(45)=	30.0	Y(45)=	-59.689
X(46)=	73.3	Y(46)=	-146.061
X(47)=	99.2	Y(47)=	-198.362
X(48)=	11.0	Y(48)=	-21.214
X(49)=	50.3	Y(49)=	-98.913
X(50)=	28.0	Y(50)=	-55.397
X(51)=	73.4	Y(51)=	-144.924
X(52)=	57.4	Y(52)=	-113.848
X(53)=	30.9	Y(53)=	-60.505
X(54)=	30.5	Y(54)=	-60.540
X(55)=	19.2	Y(55)=	-37.657
X(56)=	48.6	Y(56)=	-96.705
X(57)=	57.5	Y(57)=	-114.699
X(58)=	67.6	Y(58)=	-132.175
X(59)=	1.0	Y(59)=	-1.538
X(60)=	90.9	Y(60)=	-181.617
X(61)=	27.3	Y(61)=	-53.790
X(62)=	66.4	Y(62)=	-131.050
X(63)=	22.7	Y(63)=	-43.772
X(64)=	14.4	Y(64)=	-28.290
X(65)=	30.0	Y(65)=	-59.001
X(66)=	73.3	Y(66)=	-145.987
X(67)=	99.2	Y(67)=	-197.942
X(68)=	11.0	Y(68)=	-20.791
X(69)=	50.3	Y(69)=	-100.109
X(70)=	28.0	Y(70)=	-55.833
X(71)=	73.4	Y(71)=	-145.524
X(72)=	57.4	Y(72)=	-112.920
X(73)=	30.9	Y(73)=	-60.874
X(74)=	30.5	Y(74)=	-60.940
X(75)=	19.2	Y(75)=	-37.362
X(76)=	48.6	Y(76)=	-95.721
X(77)=	57.5	Y(77)=	-112.217
X(78)=	67.6	Y(78)=	-135.030
X(79)=	1.0	Y(79)=	-1.956
X(80)=	90.9	Y(80)=	-180.957
X(81)=	27.3	Y(81)=	-51.937
X(82)=	66.4	Y(82)=	-132.480
X(83)=	22.7	Y(83)=	-42.716
X(84)=	14.4	Y(84)=	-28.118
X(85)=	30.0	Y(85)=	-52.870
X(86)=	73.3	Y(86)=	-146.519
X(87)=	99.2	Y(87)=	-197.793
X(88)=	11.0	Y(88)=	-18.761
X(89)=	50.3	Y(89)=	-100.012
X(90)=	28.0	Y(90)=	-55.925
X(91)=	73.4	Y(91)=	-144.747
X(92)=	57.4	Y(92)=	-114.016
X(93)=	30.9	Y(93)=	-61.211

```
X( 94)=  30.5   Y( 94)=     -60.074
X( 95)=  19.2   Y( 95)=     -37.570
X( 96)=  48.6   Y( 96)=     -93.450
X( 97)=  57.5   Y( 97)=    -114.938
X( 98)=  67.6   Y( 98)=    -133.252
X( 99)=   1.0   Y( 99)=      -1.967
X(100)=  90.9   Y(100)=    -181.632
T(A0)=      1.150 T(A1)=   -1.050
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR CHI-SQUARE WITH POSITIVE REGRESSION

```
A0=    2.0 EST A0=    1.521 A1=    2.0 EST A1=    2.007
X(  1)=  95.0   Y(  1)=     192.306
X(  2)=  68.8   Y(  2)=     138.690
X(  3)=  86.2   Y(  3)=     174.387
X(  4)=  83.5   Y(  4)=     168.489
X(  5)=  59.1   Y(  5)=     123.200
X(  6)=  11.4   Y(  6)=      23.848
X(  7)=  53.0   Y(  7)=     109.032
X(  8)=  33.0   Y(  8)=      67.207
X(  9)=  20.8   Y(  9)=      42.977
X( 10)=   4.2   Y( 10)=       8.647
X( 11)=  83.9   Y( 11)=     172.301
X( 12)=  93.6   Y( 12)=     187.995
X( 13)=  31.8   Y( 13)=      65.523
X( 14)=  55.3   Y( 14)=     111.046
X( 15)=  43.3   Y( 15)=      92.008
X( 16)=  46.4   Y( 16)=      94.547
X( 17)=  86.5   Y( 17)=     177.436
X( 18)=  54.4   Y( 18)=     111.886
X( 19)=  63.4   Y( 19)=     126.886
X( 20)=  72.3   Y( 20)=     144.869
X( 21)=  95.0   Y( 21)=     193.195
X( 22)=  68.8   Y( 22)=     139.178
X( 23)=  86.2   Y( 23)=     173.378
X( 24)=  83.5   Y( 24)=     168.915
X( 25)=  59.1   Y( 25)=     119.783
X( 26)=  11.4   Y( 26)=      23.346
X( 27)=  53.0   Y( 27)=     106.433
X( 28)=  33.0   Y( 28)=      66.701
X( 29)=  20.8   Y( 29)=      46.592
X( 30)=   4.2   Y( 30)=      10.529
X( 31)=  83.9   Y( 31)=     171.785
X( 32)=  93.6   Y( 32)=     192.285
X( 33)=  31.8   Y( 33)=      64.632
X( 34)=  55.3   Y( 34)=     113.490
X( 35)=  43.3   Y( 35)=      87.826
X( 36)=  46.4   Y( 36)=      93.959
X( 37)=  86.5   Y( 37)=     174.042
X( 38)=  54.4   Y( 38)=     109.507
```

X(39)=	63.4	Y(39)=	127.301
X(40)=	72.3	Y(40)=	147.197
X(41)=	95.0	Y(41)=	193.032
X(42)=	68.8	Y(42)=	138.807
X(43)=	86.2	Y(43)=	173.778
X(44)=	83.5	Y(44)=	167.247
X(45)=	59.1	Y(45)=	122.701
X(46)=	11.4	Y(46)=	23.595
X(47)=	53.0	Y(47)=	107.923
X(48)=	33.0	Y(48)=	66.446
X(49)=	20.8	Y(49)=	47.008
X(50)=	4.2	Y(50)=	10.147
X(51)=	83.9	Y(51)=	172.236
X(52)=	93.6	Y(52)=	190.286
X(53)=	31.8	Y(53)=	63.686
X(54)=	55.3	Y(54)=	110.869
X(55)=	43.3	Y(55)=	89.795
X(56)=	46.4	Y(56)=	94.378
X(57)=	86.5	Y(57)=	173.978
X(58)=	54.4	Y(58)=	110.715
X(59)=	63.4	Y(59)=	128.383
X(60)=	72.3	Y(60)=	145.146
X(61)=	95.0	Y(61)=	192.500
X(62)=	68.8	Y(62)=	139.709
X(63)=	86.2	Y(63)=	173.492
X(64)=	83.5	Y(64)=	167.399
X(65)=	59.1	Y(65)=	119.876
X(66)=	11.4	Y(66)=	24.421
X(67)=	53.0	Y(67)=	109.569
X(68)=	33.0	Y(68)=	66.519
X(69)=	20.8	Y(69)=	43.364
X(70)=	4.2	Y(70)=	10.029
X(71)=	83.9	Y(71)=	170.123
X(72)=	93.6	Y(72)=	191.030
X(73)=	31.8	Y(73)=	65.522
X(74)=	55.3	Y(74)=	113.440
X(75)=	43.3	Y(75)=	88.243
X(76)=	46.4	Y(76)=	94.155
X(77)=	86.5	Y(77)=	174.090
X(78)=	54.4	Y(78)=	110.010
X(79)=	63.4	Y(79)=	127.666
X(80)=	72.3	Y(80)=	146.960
X(81)=	95.0	Y(81)=	190.933
X(82)=	68.8	Y(82)=	137.718
X(83)=	86.2	Y(83)=	175.999
X(84)=	83.5	Y(84)=	167.217
X(85)=	59.1	Y(85)=	121.328
X(86)=	11.4	Y(86)=	24.285
X(87)=	53.0	Y(87)=	108.047
X(88)=	33.0	Y(88)=	69.132
X(89)=	20.8	Y(89)=	42.792
X(90)=	4.2	Y(90)=	9.051
X(91)=	83.9	Y(91)=	170.118

X(92)=	93.6	Y(92)=	189.440
X(93)=	31.8	Y(93)=	68.795
X(94)=	55.3	Y(94)=	112.147
X(95)=	43.3	Y(95)=	88.139
X(96)=	46.4	Y(96)=	93.791
X(97)=	86.5	Y(97)=	175.812
X(98)=	54.4	Y(98)=	110.744
X(99)=	63.4	Y(99)=	128.332
X(100)=	72.3	Y(100)=	146.382
T(A0)=	-1.492	T(A1)=	1.398

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR CHI-SQUARE WITH HORIZONTAL REGRESSION

A0= 2.0 EST A0= 2.401 A1= 0.0 EST A1= -.007

X(1)=	66.6	Y(1)=	2.393
X(2)=	19.2	Y(2)=	1.768
X(3)=	44.4	Y(3)=	1.062
X(4)=	64.2	Y(4)=	1.749
X(5)=	70.5	Y(5)=	1.817
X(6)=	84.0	Y(6)=	.740
X(7)=	46.7	Y(7)=	2.434
X(8)=	26.7	Y(8)=	1.510
X(9)=	93.7	Y(9)=	.249
X(10)=	49.5	Y(10)=	.856
X(11)=	19.5	Y(11)=	1.840
X(12)=	46.6	Y(12)=	2.337
X(13)=	74.3	Y(13)=	2.545
X(14)=	7.1	Y(14)=	.818
X(15)=	33.3	Y(15)=	3.120
X(16)=	4.2	Y(16)=	2.703
X(17)=	66.4	Y(17)=	.492
X(18)=	85.5	Y(18)=	1.724
X(19)=	15.2	Y(19)=	2.049
X(20)=	6.3	Y(20)=	1.272
X(21)=	66.6	Y(21)=	1.564
X(22)=	19.2	Y(22)=	1.338
X(23)=	44.4	Y(23)=	.644
X(24)=	64.2	Y(24)=	1.279
X(25)=	70.5	Y(25)=	.424
X(26)=	84.0	Y(26)=	3.664
X(27)=	46.7	Y(27)=	2.681
X(28)=	26.7	Y(28)=	2.830
X(29)=	93.7	Y(29)=	.048
X(30)=	49.5	Y(30)=	3.908
X(31)=	19.5	Y(31)=	3.908
X(32)=	46.6	Y(32)=	2.762
X(33)=	74.3	Y(33)=	.937
X(34)=	7.1	Y(34)=	.811
X(35)=	33.3	Y(35)=	.762
X(36)=	4.2	Y(36)=	2.283

X(37)=	66.4	Y(37)=	2.210
X(38)=	85.5	Y(38)=	1.227
X(39)=	15.2	Y(39)=	1.038
X(40)=	6.3	Y(40)=	5.670
X(41)=	66.6	Y(41)=	5.023
X(42)=	19.2	Y(42)=	2.885
X(43)=	44.4	Y(43)=	.233
X(44)=	64.2	Y(44)=	1.240
X(45)=	70.5	Y(45)=	2.978
X(46)=	84.0	Y(46)=	2.797
X(47)=	46.7	Y(47)=	1.245
X(48)=	26.7	Y(48)=	2.040
X(49)=	93.7	Y(49)=	2.257
X(50)=	49.5	Y(50)=	1.402
X(51)=	19.5	Y(51)=	1.181
X(52)=	46.6	Y(52)=	2.485
X(53)=	74.3	Y(53)=	4.230
X(54)=	7.1	Y(54)=	2.219
X(55)=	33.3	Y(55)=	.637
X(56)=	4.2	Y(56)=	3.000
X(57)=	66.4	Y(57)=	3.382
X(58)=	85.5	Y(58)=	1.036
X(59)=	15.2	Y(59)=	.417
X(60)=	6.3	Y(60)=	.749
X(61)=	66.6	Y(61)=	2.129
X(62)=	19.2	Y(62)=	3.985
X(63)=	44.4	Y(63)=	5.085
X(64)=	64.2	Y(64)=	1.032
X(65)=	70.5	Y(65)=	2.890
X(66)=	84.0	Y(66)=	1.226
X(67)=	46.7	Y(67)=	1.159
X(68)=	26.7	Y(68)=	1.042
X(69)=	93.7	Y(69)=	.707
X(70)=	49.5	Y(70)=	.501
X(71)=	19.5	Y(71)=	2.597
X(72)=	46.6	Y(72)=	2.481
X(73)=	74.3	Y(73)=	.759
X(74)=	7.1	Y(74)=	4.979
X(75)=	33.3	Y(75)=	3.328
X(76)=	4.2	Y(76)=	.777
X(77)=	66.4	Y(77)=	.546
X(78)=	85.5	Y(78)=	3.799
X(79)=	15.2	Y(79)=	1.600
X(80)=	6.3	Y(80)=	.827
X(81)=	66.6	Y(81)=	2.552
X(82)=	19.2	Y(82)=	6.516
X(83)=	44.4	Y(83)=	1.115
X(84)=	64.2	Y(84)=	4.787
X(85)=	70.5	Y(85)=	1.571
X(86)=	84.0	Y(86)=	3.630
X(87)=	46.7	Y(87)=	2.161
X(88)=	26.7	Y(88)=	5.501
X(89)=	93.7	Y(89)=	2.116

```
X( 90)=  49.5   Y( 90)=        2.333
X( 91)=  19.5   Y( 91)=        3.150
X( 92)=  46.6   Y( 92)=        1.351
X( 93)=  74.3   Y( 93)=        1.315
X( 94)=   7.1   Y( 94)=        2.602
X( 95)=  33.3   Y( 95)=         .928
X( 96)=   4.2   Y( 96)=        4.512
X( 97)=  66.4   Y( 97)=        2.911
X( 98)=  85.5   Y( 98)=         .911
X( 99)=  15.2   Y( 99)=        1.354
X(100)=   6.3   Y(100)=        1.719
T(A0)=      1.527 T(A1)=     -1.368
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR CHI-SQUARE WITH NEGATIVE REGRESSION

```
A0=   2.0 EST A0=   2.262 A1=  -2.0 EST A1=  -2.004
X(  1)=  27.3   Y(  1)=      -51.673
X(  2)=  66.4   Y(  2)=     -131.980
X(  3)=  22.7   Y(  3)=      -41.463
X(  4)=  14.4   Y(  4)=      -28.073
X(  5)=  30.0   Y(  5)=      -59.162
X(  6)=  73.3   Y(  6)=     -144.927
X(  7)=  99.2   Y(  7)=     -196.682
X(  8)=  11.0   Y(  8)=      -21.432
X(  9)=  50.3   Y(  9)=      -98.002
X( 10)=  28.0   Y( 10)=      -52.734
X( 11)=  73.4   Y( 11)=     -146.337
X( 12)=  57.4   Y( 12)=     -112.235
X( 13)=  30.9   Y( 13)=      -57.033
X( 14)=  30.5   Y( 14)=      -58.245
X( 15)=  19.2   Y( 15)=      -36.432
X( 16)=  48.6   Y( 16)=      -95.382
X( 17)=  57.5   Y( 17)=     -113.847
X( 18)=  67.6   Y( 18)=     -132.960
X( 19)=   1.0   Y( 19)=       -1.373
X( 20)=  90.9   Y( 20)=     -180.822
X( 21)=  27.3   Y( 21)=      -53.833
X( 22)=  66.4   Y( 22)=     -130.134
X( 23)=  22.7   Y( 23)=      -40.931
X( 24)=  14.4   Y( 24)=      -26.258
X( 25)=  30.0   Y( 25)=      -57.323
X( 26)=  73.3   Y( 26)=     -146.079
X( 27)=  99.2   Y( 27)=     -197.178
X( 28)=  11.0   Y( 28)=      -17.213
X( 29)=  50.3   Y( 29)=      -99.996
X( 30)=  28.0   Y( 30)=      -53.920
X( 31)=  73.4   Y( 31)=     -143.698
X( 32)=  57.4   Y( 32)=     -114.198
X( 33)=  30.9   Y( 33)=      -60.739
X( 34)=  30.5   Y( 34)=      -60.659
```

X(35)=	19.2	Y(35)=	-34.087
X(36)=	48.6	Y(36)=	-95.294
X(37)=	57.5	Y(37)=	-114.480
X(38)=	67.6	Y(38)=	-129.578
X(39)=	1.0	Y(39)=	-.663
X(40)=	90.9	Y(40)=	-178.922
X(41)=	27.3	Y(41)=	-53.525
X(42)=	66.4	Y(42)=	-129.594
X(43)=	22.7	Y(43)=	-45.269
X(44)=	14.4	Y(44)=	-27.367
X(45)=	30.0	Y(45)=	-58.346
X(46)=	73.3	Y(46)=	-143.017
X(47)=	99.2	Y(47)=	-195.786
X(48)=	11.0	Y(48)=	-20.917
X(49)=	50.3	Y(49)=	-99.784
X(50)=	28.0	Y(50)=	-55.146
X(51)=	73.4	Y(51)=	-144.667
X(52)=	57.4	Y(52)=	-113.475
X(53)=	30.9	Y(53)=	-58.902
X(54)=	30.5	Y(54)=	-58.074
X(55)=	19.2	Y(55)=	-34.849
X(56)=	48.6	Y(56)=	-94.099
X(57)=	57.5	Y(57)=	-113.271
X(58)=	67.6	Y(58)=	-133.839
X(59)=	1.0	Y(59)=	.736
X(60)=	90.9	Y(60)=	-178.029
X(61)=	27.3	Y(61)=	-52.831
X(62)=	66.4	Y(62)=	-130.574
X(63)=	22.7	Y(63)=	-42.888
X(64)=	14.4	Y(64)=	-28.405
X(65)=	30.0	Y(65)=	-56.704
X(66)=	73.3	Y(66)=	-146.351
X(67)=	99.2	Y(67)=	-197.352
X(68)=	11.0	Y(68)=	-17.486
X(69)=	50.3	Y(69)=	-99.353
X(70)=	28.0	Y(70)=	-52.837
X(71)=	73.4	Y(71)=	-146.209
X(72)=	57.4	Y(72)=	-113.693
X(73)=	30.9	Y(73)=	-58.455
X(74)=	30.5	Y(74)=	-60.203
X(75)=	19.2	Y(75)=	-38.278
X(76)=	48.6	Y(76)=	-96.715
X(77)=	57.5	Y(77)=	-109.284
X(78)=	67.6	Y(78)=	-134.634
X(79)=	1.0	Y(79)=	1.343
X(80)=	90.9	Y(80)=	-180.478
X(81)=	27.3	Y(81)=	-53.538
X(82)=	66.4	Y(82)=	-131.390
X(83)=	22.7	Y(83)=	-43.142
X(84)=	14.4	Y(84)=	-27.297
X(85)=	30.0	Y(85)=	-59.196
X(86)=	73.3	Y(86)=	-143.866
X(87)=	99.2	Y(87)=	-196.432

```
X( 88)=  11.0    Y( 88)=     -19.620
X( 89)=  50.3    Y( 89)=     -93.280
X( 90)=  28.0    Y( 90)=     -53.762
X( 91)=  73.4    Y( 91)=    -143.080
X( 92)=  57.4    Y( 92)=    -112.419
X( 93)=  30.9    Y( 93)=     -58.225
X( 94)=  30.5    Y( 94)=     -59.481
X( 95)=  19.2    Y( 95)=     -33.858
X( 96)=  48.6    Y( 96)=     -96.436
X( 97)=  57.5    Y( 97)=    -111.302
X( 98)=  67.6    Y( 98)=    -134.035
X( 99)=   1.0    Y( 99)=      -1.032
X(100)=  90.9    Y(100)=    -181.430
T(A0)=         .944 T(A1)=     -.739
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR SKEWED NORMAL WITH POSITIVE REGRESSION

```
A0=   4.0 EST A0=   3.184 A1=   2.0 EST A1=   2.011
X(  1)=  95.0    Y(  1)=     194.578
X(  2)=  68.8    Y(  2)=     140.351
X(  3)=  86.2    Y(  3)=     176.525
X(  4)=  83.5    Y(  4)=     170.386
X(  5)=  59.1    Y(  5)=     126.270
X(  6)=  11.4    Y(  6)=      25.482
X(  7)=  53.0    Y(  7)=     111.566
X(  8)=  33.0    Y(  8)=      68.943
X(  9)=  20.8    Y(  9)=      44.813
X( 10)=   4.2    Y( 10)=       9.502
X( 11)=  83.9    Y( 11)=     175.252
X( 12)=  93.6    Y( 12)=     189.444
X( 13)=  31.8    Y( 13)=      67.632
X( 14)=  55.3    Y( 14)=     112.168
X( 15)=  43.3    Y( 15)=      95.169
X( 16)=  46.4    Y( 16)=      96.575
X( 17)=  86.5    Y( 17)=     180.371
X( 18)=  54.4    Y( 18)=     114.437
X( 19)=  63.4    Y( 19)=     127.407
X( 20)=  72.3    Y( 20)=     145.759
X( 21)=  95.0    Y( 21)=     195.781
X( 22)=  68.8    Y( 22)=     141.122
X( 23)=  86.2    Y( 23)=     174.964
X( 24)=  83.5    Y( 24)=     171.020
X( 25)=  59.1    Y( 25)=     121.729
X( 26)=  11.4    Y( 26)=      24.573
X( 27)=  53.0    Y( 27)=     107.539
X( 28)=  33.0    Y( 28)=      68.073
X( 29)=  20.8    Y( 29)=      49.660
X( 30)=   4.2    Y( 30)=      12.728
X( 31)=  83.9    Y( 31)=     174.603
X( 32)=  93.6    Y( 32)=     195.374
```

```
X( 33)=  31.8   Y( 33)=    66.256
X( 34)=  55.3   Y( 34)=   115.977
X( 35)=  43.3   Y( 35)=    89.574
X( 36)=  46.4   Y( 36)=    95.666
X( 37)=  86.5   Y( 37)=   175.671
X( 38)=  54.4   Y( 38)=   110.884
X( 39)=  63.4   Y( 39)=   128.482
X( 40)=  72.3   Y( 40)=   149.579
X( 41)=  95.0   Y( 41)=   194.846
X( 42)=  68.8   Y( 42)=   141.899
X( 43)=  86.2   Y( 43)=   175.155
X( 44)=  83.5   Y( 44)=   168.465
X( 45)=  59.1   Y( 45)=   121.869
X( 46)=  11.4   Y( 46)=    26.387
X( 47)=  53.0   Y( 47)=   112.269
X( 48)=  33.0   Y( 48)=    67.720
X( 49)=  20.8   Y( 49)=    45.399
X( 50)=   4.2   Y( 50)=    11.998
X( 51)=  83.9   Y( 51)=   172.401
X( 52)=  93.6   Y( 52)=   193.804
X( 53)=  31.8   Y( 53)=    67.631
X( 54)=  55.3   Y( 54)=   115.908
X( 55)=  43.3   Y( 55)=    90.219
X( 56)=  46.4   Y( 56)=    95.978
X( 57)=  86.5   Y( 57)=   175.752
X( 58)=  54.4   Y( 58)=   111.747
X( 59)=  63.4   Y( 59)=   129.171
X( 60)=  72.3   Y( 60)=   149.253
X( 61)=  95.0   Y( 61)=   192.488
X( 62)=  68.8   Y( 62)=   138.323
X( 63)=  86.2   Y( 63)=   178.708
X( 64)=  83.5   Y( 64)=   168.023
X( 65)=  59.1   Y( 65)=   123.893
X( 66)=  11.4   Y( 66)=    26.180
X( 67)=  53.0   Y( 67)=   110.211
X( 68)=  33.0   Y( 68)=    71.698
X( 69)=  20.8   Y( 69)=    44.519
X( 70)=   4.2   Y( 70)=    10.379
X( 71)=  83.9   Y( 71)=   172.394
X( 72)=  93.6   Y( 72)=   191.685
X( 73)=  31.8   Y( 73)=    71.909
X( 74)=  55.3   Y( 74)=   114.074
X( 75)=  43.3   Y( 75)=    90.062
X( 76)=  46.4   Y( 76)=    95.386
X( 77)=  86.5   Y( 77)=   178.271
X( 78)=  54.4   Y( 78)=   112.863
X( 79)=  63.4   Y( 79)=   130.251
X( 80)=  72.3   Y( 80)=   148.426
X( 81)=  95.0   Y( 81)=   194.699
X( 82)=  68.8   Y( 82)=   141.405
X( 83)=  86.2   Y( 83)=   175.105
X( 84)=  83.5   Y( 84)=   170.777
X( 85)=  59.1   Y( 85)=   122.077
```

```
X( 86)=  11.4    Y( 86)=        24.945
X( 87)=  53.0    Y( 87)=       110.755
X( 88)=  33.0    Y( 88)=        69.418
X( 89)=  20.8    Y( 89)=        42.708
X( 90)=   4.2    Y( 90)=        10.753
X( 91)=  83.9    Y( 91)=       171.711
X( 92)=  93.6    Y( 92)=       191.820
X( 93)=  31.8    Y( 93)=        68.508
X( 94)=  55.3    Y( 94)=       112.885
X( 95)=  43.3    Y( 95)=        92.282
X( 96)=  46.4    Y( 96)=        97.924
X( 97)=  86.5    Y( 97)=       174.664
X( 98)=  54.4    Y( 98)=       112.540
X( 99)=  63.4    Y( 99)=       131.014
X(100)=  72.3    Y(100)=       147.646
T(A0)=     -1.636 T(A1)=       1.619
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR SKEWED NORMAL WITH HORIZONTAL REGRESSION

```
A0=    4.0 EST A0=    4.464 A1=    0.0 EST A1=    -.005
X(  1)=  66.6    Y(  1)=          1.966
X(  2)=  19.2    Y(  2)=          3.058
X(  3)=  44.4    Y(  3)=          1.521
X(  4)=  64.2    Y(  4)=          6.392
X(  5)=  70.5    Y(  5)=          5.094
X(  6)=  84.0    Y(  6)=          5.295
X(  7)=  46.7    Y(  7)=           .444
X(  8)=  26.7    Y(  8)=          6.705
X(  9)=  93.7    Y(  9)=          6.704
X( 10)=  49.5    Y( 10)=          5.203
X( 11)=  19.5    Y( 11)=          2.494
X( 12)=  46.6    Y( 12)=          2.273
X( 13)=  74.3    Y( 13)=          2.186
X( 14)=   7.1    Y( 14)=          4.545
X( 15)=  33.3    Y( 15)=          4.442
X( 16)=   4.2    Y( 16)=          2.975
X( 17)=  66.4    Y( 17)=          2.665
X( 18)=  85.5    Y( 18)=          8.886
X( 19)=  15.2    Y( 19)=          8.098
X( 20)=   6.3    Y( 20)=          5.369
X( 21)=  66.6    Y( 21)=          3.899
X( 22)=  19.2    Y( 22)=          2.676
X( 23)=  44.4    Y( 23)=          1.928
X( 24)=  64.2    Y( 24)=          1.204
X( 25)=  70.5    Y( 25)=          1.847
X( 26)=  84.0    Y( 26)=          5.021
X( 27)=  46.7    Y( 27)=          3.212
X( 28)=  26.7    Y( 28)=          7.335
X( 29)=  93.7    Y( 29)=          3.377
X( 30)=  49.5    Y( 30)=          2.850
```

```
X( 31)=  19.5   Y( 31)=         2.143
X( 32)=  46.6   Y( 32)=         5.207
X( 33)=  74.3   Y( 33)=         2.708
X( 34)=   7.1   Y( 34)=         8.050
X( 35)=  33.3   Y( 35)=         1.729
X( 36)=   4.2   Y( 36)=         3.700
X( 37)=  66.4   Y( 37)=         4.022
X( 38)=  85.5   Y( 38)=         5.122
X( 39)=  15.2   Y( 39)=         4.918
X( 40)=   6.3   Y( 40)=         9.899
X( 41)=  66.6   Y( 41)=         2.794
X( 42)=  19.2   Y( 42)=         7.807
X( 43)=  44.4   Y( 43)=         3.511
X( 44)=  64.2   Y( 44)=         6.349
X( 45)=  70.5   Y( 45)=         4.373
X( 46)=  84.0   Y( 46)=         8.681
X( 47)=  46.7   Y( 47)=         4.310
X( 48)=  26.7   Y( 48)=         4.615
X( 49)=  93.7   Y( 49)=         5.695
X( 50)=  49.5   Y( 50)=         3.172
X( 51)=  19.5   Y( 51)=         3.115
X( 52)=  46.6   Y( 52)=         4.987
X( 53)=  74.3   Y( 53)=         2.479
X( 54)=   7.1   Y( 54)=         7.466
X( 55)=  33.3   Y( 55)=         5.404
X( 56)=   4.2   Y( 56)=         2.449
X( 57)=  66.4   Y( 57)=         3.177
X( 58)=  85.5   Y( 58)=         3.733
X( 59)=  15.2   Y( 59)=         5.424
X( 60)=   6.3   Y( 60)=         2.287
X( 61)=  66.6   Y( 61)=         6.740
X( 62)=  19.2   Y( 62)=         2.120
X( 63)=  44.4   Y( 63)=         2.320
X( 64)=  64.2   Y( 64)=         3.663
X( 65)=  70.5   Y( 65)=         3.730
X( 66)=  84.0   Y( 66)=         1.816
X( 67)=  46.7   Y( 67)=         4.980
X( 68)=  26.7   Y( 68)=         5.874
X( 69)=  93.7   Y( 69)=         1.602
X( 70)=  49.5   Y( 70)=         4.934
X( 71)=  19.5   Y( 71)=         7.781
X( 72)=  46.6   Y( 72)=         5.192
X( 73)=  74.3   Y( 73)=         4.095
X( 74)=   7.1   Y( 74)=         3.878
X( 75)=  33.3   Y( 75)=         2.854
X( 76)=   4.2   Y( 76)=         4.484
X( 77)=  66.4   Y( 77)=         1.932
X( 78)=  85.5   Y( 78)=         2.563
X( 79)=  15.2   Y( 79)=         2.192
X( 80)=   6.3   Y( 80)=         5.072
X( 81)=  66.6   Y( 81)=         7.411
X( 82)=  19.2   Y( 82)=         4.902
X( 83)=  44.4   Y( 83)=         5.086
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X( 84)=  64.2   Y( 84)=      1.722
X( 85)=  70.5   Y( 85)=      2.966
X( 86)=  84.0   Y( 86)=      7.806
X( 87)=  46.7   Y( 87)=      1.886
X( 88)=  26.7   Y( 88)=      4.257
X( 89)=  93.7   Y( 89)=      5.657
X( 90)=  49.5   Y( 90)=      1.883
X( 91)=  19.5   Y( 91)=      2.701
X( 92)=  46.6   Y( 92)=      1.332
X( 93)=  74.3   Y( 93)=      7.215
X( 94)=   7.1   Y( 94)=      4.006
X( 95)=  33.3   Y( 95)=      1.719
X( 96)=   4.2   Y( 96)=      8.828
X( 97)=  66.4   Y( 97)=      3.148
X( 98)=  85.5   Y( 98)=      5.359
X( 99)=  15.2   Y( 99)=      2.726
X(100)=   6.3   Y(100)=      5.794
T(A0)=      1.151 T(A1)=     -.672
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR SKEWED NORMAL WITH NEGATIVE REGRESSION

```
A0=   4.0 EST A0=   3.807 A1=  -2.0 EST A1=  -2.015
X(  1)=  27.3   Y(  1)=    -49.598
X(  2)=  66.4   Y(  2)=   -130.061
X(  3)=  22.7   Y(  3)=    -43.118
X(  4)=  14.4   Y(  4)=    -26.451
X(  5)=  30.0   Y(  5)=    -55.667
X(  6)=  73.3   Y(  6)=   -143.470
X(  7)=  99.2   Y(  7)=   -193.014
X(  8)=  11.0   Y(  8)=    -16.576
X(  9)=  50.3   Y(  9)=    -94.354
X( 10)=  28.0   Y( 10)=    -50.343
X( 11)=  73.4   Y( 11)=   -143.053
X( 12)=  57.4   Y( 12)=   -111.612
X( 13)=  30.9   Y( 13)=    -56.630
X( 14)=  30.5   Y( 14)=    -54.472
X( 15)=  19.2   Y( 15)=    -34.594
X( 16)=  48.6   Y( 16)=    -92.735
X( 17)=  57.5   Y( 17)=   -110.137
X( 18)=  67.6   Y( 18)=   -133.746
X( 19)=   1.0   Y( 19)=      3.912
X( 20)=  90.9   Y( 20)=   -180.694
X( 21)=  27.3   Y( 21)=    -54.045
X( 22)=  66.4   Y( 22)=   -130.389
X( 23)=  22.7   Y( 23)=    -39.423
X( 24)=  14.4   Y( 24)=    -26.571
X( 25)=  30.0   Y( 25)=    -57.573
X( 26)=  73.3   Y( 26)=   -146.226
X( 27)=  99.2   Y( 27)=   -197.494
X( 28)=  11.0   Y( 28)=    -15.497
```

X(29)=	50.3	Y(29)=	-100.166
X(30)=	28.0	Y(30)=	-54.324
X(31)=	73.4	Y(31)=	-143.590
X(32)=	57.4	Y(32)=	-114.367
X(33)=	30.9	Y(33)=	-61.021
X(34)=	30.5	Y(34)=	-60.756
X(35)=	19.2	Y(35)=	-32.695
X(36)=	48.6	Y(36)=	-95.699
X(37)=	57.5	Y(37)=	-114.627
X(38)=	67.6	Y(38)=	-127.430
X(39)=	1.0	Y(39)=	-1.002
X(40)=	90.9	Y(40)=	-179.033
X(41)=	27.3	Y(41)=	-53.228
X(42)=	66.4	Y(42)=	-130.966
X(43)=	22.7	Y(43)=	-43.212
X(44)=	14.4	Y(44)=	-28.518
X(45)=	30.0	Y(45)=	-56.364
X(46)=	73.3	Y(46)=	-146.423
X(47)=	99.2	Y(47)=	-197.631
X(48)=	11.0	Y(48)=	-15.947
X(49)=	50.3	Y(49)=	-99.674
X(50)=	28.0	Y(50)=	-52.660
X(51)=	73.4	Y(51)=	-146.375
X(52)=	57.4	Y(52)=	-113.985
X(53)=	30.9	Y(53)=	-58.053
X(54)=	30.5	Y(54)=	-60.422
X(55)=	19.2	Y(55)=	-38.314
X(56)=	48.6	Y(56)=	-96.853
X(57)=	57.5	Y(57)=	-107.093
X(58)=	67.6	Y(58)=	-134.793
X(59)=	1.0	Y(59)=	1.743
X(60)=	90.9	Y(60)=	-180.814
X(61)=	27.3	Y(61)=	-53.820
X(62)=	66.4	Y(62)=	-131.742
X(63)=	22.7	Y(63)=	-43.529
X(64)=	14.4	Y(64)=	-27.663
X(65)=	30.0	Y(65)=	-59.417
X(66)=	73.3	Y(66)=	-144.082
X(67)=	99.2	Y(67)=	-196.838
X(68)=	11.0	Y(68)=	-19.983
X(69)=	50.3	Y(69)=	-90.526
X(70)=	28.0	Y(70)=	-54.152
X(71)=	73.4	Y(71)=	-142.233
X(72)=	57.4	Y(72)=	-112.782
WX 63G7	30G8	XX 63G7	-46G433
X(74)=	30.5	Y(74)=	-59.849
X(75)=	19.2	Y(75)=	-32.301
X(76)=	48.6	Y(76)=	-96.647
X(77)=	57.5	Y(77)=	-110.480
X(78)=	67.6	Y(78)=	-134.340
X(79)=	1.0	Y(79)=	-1.293
X(80)=	90.9	Y(80)=	-181.536
X(81)=	27.3	Y(81)=	-50.503

```
X( 82)=  66.4    Y( 82)=     -128.120
X( 83)=  22.7    Y( 83)=      -34.550
X( 84)=  14.4    Y( 84)=      -24.318
X( 85)=  30.0    Y( 85)=      -53.537
X( 86)=  73.3    Y( 86)=     -141.918
X( 87)=  99.2    Y( 87)=     -192.124
X( 88)=  11.0    Y( 88)=      -18.569
X( 89)=  50.3    Y( 89)=      -93.098
X( 90)=  28.0    Y( 90)=      -53.813
X( 91)=  73.4    Y( 91)=     -140.365
X( 92)=  57.4    Y( 92)=     -111.926
X( 93)=  30.9    Y( 93)=      -59.254
X( 94)=  30.5    Y( 94)=      -59.601
X( 95)=  19.2    Y( 95)=      -29.939
X( 96)=  48.6    Y( 96)=      -94.397
X( 97)=  57.5    Y( 97)=     -111.607
X( 98)=  67.6    Y( 98)=     -132.129
X( 99)=   1.0    Y( 99)=        5.680
X(100)=  90.9    Y(100)=     -179.190
T(A0)=       -.409 T(A1)=    -1.643
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=1) WITH POSITIVE REGRESSION

```
A0=    1.0 EST A0=      .913 A1=    2.0 EST A1=    2.000
X(  1)=  95.0    Y(  1)=      189.609
X(  2)=  68.8    Y(  2)=      138.052
X(  3)=  86.2    Y(  3)=      171.730
X(  4)=  83.5    Y(  4)=      168.083
X(  5)=  59.1    Y(  5)=      119.751
X(  6)=  11.4    Y(  6)=       23.375
X(  7)=  53.0    Y(  7)=      106.993
X(  8)=  33.0    Y(  8)=       68.647
X(  9)=  20.8    Y(  9)=       42.976
X( 10)=   4.2    Y( 10)=        8.421
X( 11)=  83.9    Y( 11)=      168.220
X( 12)=  93.6    Y( 12)=      187.615
X( 13)=  31.8    Y( 13)=       63.444
X( 14)=  55.3    Y( 14)=      111.150
X( 15)=  43.3    Y( 15)=       86.970
X( 16)=  46.4    Y( 16)=       93.945
X( 17)=  86.5    Y( 17)=      175.716
X( 18)=  54.4    Y( 18)=      108.722
X( 19)=  63.4    Y( 19)=      128.403
X( 20)=  72.3    Y( 20)=      146.199
X( 21)=  95.0    Y( 21)=      190.360
X( 22)=  68.8    Y( 22)=      139.371
X( 23)=  86.2    Y( 23)=      172.217
X( 24)=  83.5    Y( 24)=      168.338
X( 25)=  59.1    Y( 25)=      118.374
X( 26)=  11.4    Y( 26)=       23.966
```

X(27)=	53.0	Y(27)=	108.152
X(28)=	33.0	Y(28)=	65.974
X(29)=	20.8	Y(29)=	40.071
X(30)=	4.2	Y(30)=	9.884
X(31)=	83.9	Y(31)=	169.651
X(32)=	93.6	Y(32)=	187.614
X(33)=	31.8	Y(33)=	64.612
X(34)=	55.3	Y(34)=	110.085
X(35)=	43.3	Y(35)=	86.273
X(36)=	46.4	Y(36)=	93.216
X(37)=	86.5	Y(37)=	171.555
X(38)=	54.4	Y(38)=	108.729
X(39)=	63.4	Y(39)=	127.178
X(40)=	72.3	Y(40)=	145.342
X(41)=	95.0	Y(41)=	192.862
X(42)=	68.8	Y(42)=	139.176
X(43)=	86.2	Y(43)=	172.126
X(44)=	83.5	Y(44)=	168.151
X(45)=	59.1	Y(45)=	119.891
X(46)=	11.4	Y(46)=	23.987
X(47)=	53.0	Y(47)=	108.477
X(48)=	33.0	Y(48)=	67.403
X(49)=	20.8	Y(49)=	41.404
X(50)=	4.2	Y(50)=	8.921
X(51)=	83.9	Y(51)=	168.192
X(52)=	93.6	Y(52)=	189.459
X(53)=	31.8	Y(53)=	63.561
X(54)=	55.3	Y(54)=	111.938
X(55)=	43.3	Y(55)=	87.830
X(56)=	46.4	Y(56)=	95.277
X(57)=	86.5	Y(57)=	171.920
X(58)=	54.4	Y(58)=	110.198
X(59)=	63.4	Y(59)=	128.551
X(60)=	72.3	Y(60)=	145.419
X(61)=	95.0	Y(61)=	191.443
X(62)=	68.8	Y(62)=	139.061
X(63)=	86.2	Y(63)=	172.115
X(64)=	83.5	Y(64)=	168.044
X(65)=	59.1	Y(65)=	119.489
X(66)=	11.4	Y(66)=	23.088
X(67)=	53.0	Y(67)=	106.883
X(68)=	33.0	Y(68)=	68.912
X(69)=	20.8	Y(69)=	41.818
X(70)=	4.2	Y(70)=	9.038
X(71)=	83.9	Y(71)=	168.813
X(72)=	93.6	Y(72)=	188.384
X(73)=	31.8	Y(73)=	66.590
X(74)=	55.3	Y(74)=	111.871
X(75)=	43.3	Y(75)=	87.467
X(76)=	46.4	Y(76)=	93.419
X(77)=	86.5	Y(77)=	174.365
X(78)=	54.4	Y(78)=	109.747
X(79)=	63.4	Y(79)=	128.004

```
X( 80)=   72.3    Y( 80)=      145.576
X( 81)=   95.0    Y( 81)=      190.104
X( 82)=   68.8    Y( 82)=      139.026
X( 83)=   86.2    Y( 83)=      173.184
WX 73G7   72G4    XX 73G7      156G016
X( 85)=   59.1    Y( 85)=      119.166
X( 86)=   11.4    Y( 86)=       24.269
X( 87)=   53.0    Y( 87)=      106.368
X( 88)=   33.0    Y( 88)=       66.502
X( 89)=   20.8    Y( 89)=       43.311
X( 90)=    4.2    Y( 90)=        9.235
X( 91)=   83.9    Y( 91)=      169.514
X( 92)=   93.6    Y( 92)=      188.389
X( 93)=   31.8    Y( 93)=       63.699
X( 94)=   55.3    Y( 94)=      111.884
X( 95)=   43.3    Y( 95)=       88.184
X( 96)=   46.4    Y( 96)=       93.640
X( 97)=   86.5    Y( 97)=      173.890
X( 98)=   54.4    Y( 98)=      110.376
X( 99)=   63.4    Y( 99)=      127.537
X(100)=   72.3    Y(100)=      145.813
T(A0)=       -.410 T(A1)=      -.067


COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=1) WITH HORIZONTAL REGRESSION


A0=    1.0 EST A0=    1.037 A1=    0.0 EST A1=    -.000
X(  1)=   66.6    Y(  1)=        1.978
X(  2)=   19.2    Y(  2)=        2.253
X(  3)=   44.4    Y(  3)=         .963
X(  4)=   64.2    Y(  4)=        1.948
X(  5)=   70.5    Y(  5)=        2.048
X(  6)=   84.0    Y(  6)=        1.104
X(  7)=   46.7    Y(  7)=        -.045
X(  8)=   26.7    Y(  8)=        2.440
X(  9)=   93.7    Y(  9)=         .401
X( 10)=   49.5    Y( 10)=         .677
X( 11)=   19.5    Y( 11)=        1.109
X( 12)=   46.6    Y( 12)=        -.463
X( 13)=   74.3    Y( 13)=        2.797
X( 14)=    7.1    Y( 14)=         .734
X( 15)=   33.3    Y( 15)=        1.187
X( 16)=    4.2    Y( 16)=        2.994
X( 17)=   66.4    Y( 17)=       -1.002
X( 18)=   85.5    Y( 18)=        1.035
X( 19)=   15.2    Y( 19)=         .948
X( 20)=    6.3    Y( 20)=        -.423
X( 21)=   66.6    Y( 21)=         .666
X( 22)=   19.2    Y( 22)=        1.597
X( 23)=   44.4    Y( 23)=         .163
X( 24)=   64.2    Y( 24)=        1.204
```

X(25)=	70.5	Y(25)=	1.560
X(26)=	84.0	Y(26)=	3.071
X(27)=	46.7	Y(27)=	.578
X(28)=	26.7	Y(28)=	1.920
X(29)=	93.7	Y(29)=	-.062
X(30)=	49.5	Y(30)=	.469
X(31)=	19.5	Y(31)=	1.357
X(32)=	46.6	Y(32)=	2.439
X(33)=	74.3	Y(33)=	2.557
X(34)=	7.1	Y(34)=	.550
X(35)=	33.3	Y(35)=	1.258
X(36)=	4.2	Y(36)=	1.522
X(37)=	66.4	Y(37)=	.181
X(38)=	85.5	Y(38)=	3.074
X(39)=	15.2	Y(39)=	-.956
X(40)=	6.3	Y(40)=	1.928
X(41)=	66.6	Y(41)=	.567
X(42)=	19.2	Y(42)=	-.197
X(43)=	44.4	Y(43)=	1.472
X(44)=	64.2	Y(44)=	1.417
X(45)=	70.5	Y(45)=	-.522
X(46)=	84.0	Y(46)=	1.492
X(47)=	46.7	Y(47)=	2.303
X(48)=	26.7	Y(48)=	.749
X(49)=	93.7	Y(49)=	.670
X(50)=	49.5	Y(50)=	.906
X(51)=	19.5	Y(51)=	2.298
X(52)=	46.6	Y(52)=	.684
X(53)=	74.3	Y(53)=	.906
X(54)=	7.1	Y(54)=	-.196
X(55)=	33.3	Y(55)=	1.216
X(56)=	4.2	Y(56)=	.983
X(57)=	66.4	Y(57)=	1.946
X(58)=	85.5	Y(58)=	-.055
X(59)=	15.2	Y(59)=	.817
VW 40.6	4.3	WW 40.6	2.303
X(62)=	19.2	Y(62)=	1.087
X(63)=	44.4	Y(63)=	.861
X(64)=	64.2	Y(64)=	.710
X(65)=	70.5	Y(65)=	1.474
X(66)=	84.0	Y(66)=	1.994
X(67)=	46.7	Y(67)=	2.108
X(68)=	26.7	Y(68)=	1.658
X(69)=	93.7	Y(69)=	.483
X(70)=	49.5	Y(70)=	2.423
X(71)=	19.5	Y(71)=	1.319
X(72)=	46.6	Y(72)=	1.009
X(73)=	74.3	Y(73)=	-.664
X(74)=	7.1	Y(74)=	.136
X(75)=	33.3	Y(75)=	-.746
X(76)=	4.2	Y(76)=	.524
X(77)=	66.4	Y(77)=	.791
X(78)=	85.5	Y(78)=	.893

```
X( 79)=   15.2    Y( 79)=        1.670
X( 80)=    6.3    Y( 80)=        -.037
X( 81)=   66.6    Y( 81)=        -.390
X( 82)=   19.2    Y( 82)=         .452
X( 83)=   44.4    Y( 83)=        -.669
X( 84)=   64.2    Y( 84)=        1.083
X( 85)=   70.5    Y( 85)=        1.551
X( 86)=   84.0    Y( 86)=         .575
X( 87)=   46.7    Y( 87)=         .993
X( 88)=   26.7    Y( 88)=        2.647
X( 89)=   93.7    Y( 89)=        1.376
X( 90)=   49.5    Y( 90)=         .021
X( 91)=   19.5    Y( 91)=         .420
X( 92)=   46.6    Y( 92)=         .415
X( 93)=   74.3    Y( 93)=        -.155
X( 94)=    7.1    Y( 94)=         .550
X( 95)=   33.3    Y( 95)=         .370
X( 96)=    4.2    Y( 96)=        1.145
X( 97)=   66.4    Y( 97)=        2.716
X( 98)=   85.5    Y( 98)=        -.077
X( 99)=   15.2    Y( 99)=        1.603
X(100)=    6.3    Y(100)=        1.599
T(A0)=         .194 T(A1)=       -.159
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=1) WITH NEGATIVE REGRESSION

```
A0=    1.0 EST A0=     1.007 A1=  -2.0 EST A1=   -1.999
X(  1)=   27.3    Y(  1)=      -52.967
X(  2)=   66.4    Y(  2)=     -135.245
X(  3)=   22.7    Y(  3)=      -44.887
X(  4)=   14.4    Y(  4)=      -28.454
X(  5)=   30.0    Y(  5)=      -60.105
X(  6)=   73.3    Y(  6)=     -145.602
X(  7)=   99.2    Y(  7)=     -197.903
X(  8)=   11.0    Y(  8)=      -20.769
X(  9)=   50.3    Y(  9)=      -98.560
X( 10)=   28.0    Y( 10)=      -54.236
X( 11)=   73.4    Y( 11)=     -145.557
X( 12)=   57.4    Y( 12)=     -113.482
X( 13)=   30.9    Y( 13)=      -59.972
X( 14)=   30.5    Y( 14)=      -60.387
X( 15)=   19.2    Y( 15)=      -36.887
X( 16)=   48.6    Y( 16)=      -96.831
X( 17)=   57.5    Y( 17)=     -113.981
X( 18)=   67.6    Y( 18)=     -134.895
X( 19)=    1.0    Y( 19)=       -1.934
X( 20)=   90.9    Y( 20)=     -180.658
X( 21)=   27.3    Y( 21)=      -53.627
X( 22)=   66.4    Y( 22)=     -133.377
X( 23)=   22.7    Y( 23)=      -43.891
```

X(24)=	14.4	Y(24)=	-26.730
X(25)=	30.0	Y(25)=	-59.054
X(26)=	73.3	Y(26)=	-144.622
X(27)=	99.2	Y(27)=	-196.395
X(28)=	11.0	Y(28)=	-22.134
X(29)=	50.3	Y(29)=	-99.197
X(30)=	28.0	Y(30)=	-56.544
X(31)=	73.4	Y(31)=	-146.937
X(32)=	57.4	Y(32)=	-113.334
X(33)=	30.9	Y(33)=	-62.696
X(34)=	30.5	Y(34)=	-59.383
X(35)=	19.2	Y(35)=	-36.555
X(36)=	48.6	Y(36)=	-96.572
X(37)=	57.5	Y(37)=	-113.193
X(38)=	67.6	Y(38)=	-133.979
X(39)=	1.0	Y(39)=	-1.290
X(40)=	90.9	Y(40)=	-180.686
X(41)=	27.3	Y(41)=	-52.974
X(42)=	66.4	Y(42)=	-131.581
X(43)=	22.7	Y(43)=	-42.873
X(44)=	14.4	Y(44)=	-26.409
X(45)=	30.0	Y(45)=	-59.350
X(46)=	73.3	Y(46)=	-144.457
X(47)=	99.2	Y(47)=	-197.688
X(48)=	11.0	Y(48)=	-19.803
X(49)=	50.3	Y(49)=	-97.164
X(50)=	28.0	Y(50)=	-55.729
X(51)=	73.4	Y(51)=	-145.259
X(52)=	57.4	Y(52)=	-114.714
X(53)=	30.9	Y(53)=	-60.054
X(54)=	30.5	Y(54)=	-60.639
X(55)=	19.2	Y(55)=	-36.628
X(56)=	48.6	Y(56)=	-97.382
X(57)=	57.5	Y(57)=	-113.661
X(58)=	67.6	Y(58)=	-135.025
X(59)=	1.0	Y(59)=	-.833
X(60)=	90.9	Y(60)=	-179.647
X(61)=	27.3	Y(61)=	-53.945
X(62)=	66.4	Y(62)=	-131.086
X(63)=	22.7	Y(63)=	-44.393
X(64)=	14.4	Y(64)=	-29.424
X(65)=	30.0	Y(65)=	-61.339
X(66)=	73.3	Y(66)=	-146.900
X(67)=	99.2	Y(67)=	-196.065
X(68)=	11.0	Y(68)=	-19.595
X(69)=	50.3	Y(69)=	-99.850
X(70)=	28.0	Y(70)=	-55.790
X(71)=	73.4	Y(71)=	-145.175
X(72)=	57.4	Y(72)=	-112.964
X(73)=	30.9	Y(73)=	-63.118
X(74)=	30.5	Y(74)=	-60.997
X(75)=	19.2	Y(75)=	-34.761
X(76)=	48.6	Y(76)=	-95.769

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X( 77)=  57.5   Y( 77)=    -113.783
X( 78)=  67.6   Y( 78)=    -135.361
X( 79)=   1.0   Y( 79)=      -1.864
X( 80)=  90.9   Y( 80)=    -181.851
X( 81)=  27.3   Y( 81)=     -51.484
X( 82)=  66.4   Y( 82)=    -131.321
X( 83)=  22.7   Y( 83)=     -44.523
X( 84)=  14.4   Y( 84)=     -28.650
X( 85)=  30.0   Y( 85)=     -58.862
X( 86)=  73.3   Y( 86)=    -145.919
X( 87)=  99.2   Y( 87)=    -196.780
X( 88)=  11.0   Y( 88)=     -21.206
X( 89)=  50.3   Y( 89)=     -99.557
X( 90)=  28.0   Y( 90)=     -53.793
X( 91)=  73.4   Y( 91)=    -144.673
X( 92)=  57.4   Y( 92)=    -114.159
X( 93)=  30.9   Y( 93)=     -61.209
X( 94)=  30.5   Y( 94)=     -59.184
X( 95)=  19.2   Y( 95)=     -36.244
X( 96)=  48.6   Y( 96)=     -95.748
X( 97)=  57.5   Y( 97)=    -113.458
X( 98)=  67.6   Y( 98)=    -132.932
X( 99)=   1.0   Y( 99)=       -.531
X(100)=  90.9   Y(100)=    -179.814
T(A0)=       .034 T(A1)=       .341
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=2) WITH POSITIVE REGRESSION

```
A0=    2.0 EST A0=    1.586 A1=    2.0 EST A1=    2.007
X(  1)=  95.0   Y(  1)=     193.313
X(  2)=  68.8   Y(  2)=     140.246
X(  3)=  86.2   Y(  3)=     174.050
X(  4)=  83.5   Y(  4)=     167.004
X(  5)=  59.1   Y(  5)=     119.588
X(  6)=  11.4   Y(  6)=      26.283
X(  7)=  53.0   Y(  7)=     109.969
X(  8)=  33.0   Y(  8)=      66.524
X(  9)=  20.8   Y(  9)=      42.430
X( 10)=   4.2   Y( 10)=       8.967
X( 11)=  83.9   Y( 11)=     175.214
X( 12)=  93.6   Y( 12)=     188.251
X( 13)=  31.8   Y( 13)=      64.759
X( 14)=  55.3   Y( 14)=     116.017
X( 15)=  43.3   Y( 15)=      88.106
X( 16)=  46.4   Y( 16)=      95.904
X( 17)=  86.5   Y( 17)=     174.894
X( 18)=  54.4   Y( 18)=     106.353
X( 19)=  63.4   Y( 19)=     130.564
X( 20)=  72.3   Y( 20)=     144.804
X( 21)=  95.0   Y( 21)=     190.555
```

X(22)=	68.8	Y(22)=	140.096
X(23)=	86.2	Y(23)=	176.108
X(24)=	83.5	Y(24)=	170.870
X(25)=	59.1	Y(25)=	118.863
X(26)=	11.4	Y(26)=	26.566
X(27)=	53.0	Y(27)=	108.859
X(28)=	33.0	Y(28)=	67.623
X(29)=	20.8	Y(29)=	43.337
X(30)=	4.2	Y(30)=	7.281
X(31)=	83.9	Y(31)=	166.536
X(32)=	93.6	Y(32)=	190.181
X(33)=	31.8	Y(33)=	64.897
X(34)=	55.3	Y(34)=	113.963
X(35)=	43.3	Y(35)=	87.460
X(36)=	46.4	Y(36)=	96.267
X(37)=	86.5	Y(37)=	173.864
X(38)=	54.4	Y(38)=	111.532
X(39)=	63.4	Y(39)=	133.550
X(40)=	72.3	Y(40)=	147.198
X(41)=	95.0	Y(41)=	193.957
X(42)=	68.8	Y(42)=	142.107
X(43)=	86.2	Y(43)=	174.326
X(44)=	83.5	Y(44)=	170.896
X(45)=	59.1	Y(45)=	122.297
X(46)=	11.4	Y(46)=	25.008
X(47)=	53.0	Y(47)=	105.909
X(48)=	33.0	Y(48)=	70.881
X(49)=	20.8	Y(49)=	42.403
X(50)=	4.2	Y(50)=	9.755
X(51)=	83.9	Y(51)=	170.018
X(52)=	93.6	Y(52)=	186.272
X(53)=	31.8	Y(53)=	69.195
X(54)=	55.3	Y(54)=	112.069
X(55)=	43.3	Y(55)=	88.974
X(56)=	46.4	Y(56)=	98.789
X(57)=	86.5	Y(57)=	170.994
X(58)=	54.4	Y(58)=	110.870
X(59)=	63.4	Y(59)=	128.696
X(60)=	72.3	Y(60)=	143.753
X(61)=	95.0	Y(61)=	189.519
X(62)=	68.8	Y(62)=	142.277
X(63)=	86.2	Y(63)=	176.704
X(64)=	83.5	Y(64)=	167.083
X(65)=	59.1	Y(65)=	119.891
X(66)=	11.4	Y(66)=	25.610
X(67)=	53.0	Y(67)=	109.119
X(68)=	33.0	Y(68)=	68.299
X(69)=	20.8	Y(69)=	39.629
X(70)=	4.2	Y(70)=	8.390
X(71)=	83.9	Y(71)=	169.661
X(72)=	93.6	Y(72)=	190.522
X(73)=	31.8	Y(73)=	63.654
X(74)=	55.3	Y(74)=	110.336

```
X( 75)=  43.3   Y( 75)=       88.648
X( 76)=  46.4   Y( 76)=       97.471
X( 77)=  86.5   Y( 77)=      174.284
X( 78)=  54.4   Y( 78)=      112.368
X( 79)=  63.4   Y( 79)=      128.002
X( 80)=  72.3   Y( 80)=      150.467
X( 81)=  95.0   Y( 81)=      195.242
X( 82)=  68.8   Y( 82)=      136.607
X( 83)=  86.2   Y( 83)=      175.243
X( 84)=  83.5   Y( 84)=      169.429
X( 85)=  59.1   Y( 85)=      119.645
X( 86)=  11.4   Y( 86)=       24.372
X( 87)=  53.0   Y( 87)=      108.490
X( 88)=  33.0   Y( 88)=       65.877
X( 89)=  20.8   Y( 89)=       41.015
X( 90)=   4.2   Y( 90)=        9.184
X( 91)=  83.9   Y( 91)=      165.463
X( 92)=  93.6   Y( 92)=      190.932
X( 93)=  31.8   Y( 93)=       66.272
X( 94)=  55.3   Y( 94)=      108.762
X( 95)=  43.3   Y( 95)=       90.482
X( 96)=  46.4   Y( 96)=       94.313
X( 97)=  86.5   Y( 97)=      171.735
X( 98)=  54.4   Y( 98)=      110.026
X( 99)=  63.4   Y( 99)=      129.469
X(100)=  72.3   Y(100)=      145.341
T(A0)=      -.845 T(A1)=        .861


COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=2) WITH HORIZONTAL REGRESSION


A0=   2.0 EST A0=   2.879 A1=   0.0 EST A1=   -.017
X(  1)=  66.6   Y(  1)=         -.875
X(  2)=  19.2   Y(  2)=        3.497
X(  3)=  44.4   Y(  3)=        -.458
X(  4)=  64.2   Y(  4)=        4.955
X(  5)=  70.5   Y(  5)=        3.360
X(  6)=  84.0   Y(  6)=         .495
X(  7)=  46.7   Y(  7)=        2.020
X(  8)=  26.7   Y(  8)=        7.616
X(  9)=  93.7   Y(  9)=       -1.037
X( 10)=  49.5   Y( 10)=        3.738
X( 11)=  19.5   Y( 11)=        3.625
X( 12)=  46.6   Y( 12)=         .302
X( 13)=  74.3   Y( 13)=        -.549
X( 14)=   7.1   Y( 14)=         .748
X( 15)=  33.3   Y( 15)=        3.877
X( 16)=   4.2   Y( 16)=        2.516
X( 17)=  66.4   Y( 17)=        2.345
X( 18)=  85.5   Y( 18)=        3.045
X( 19)=  15.2   Y( 19)=         .295
```

X(20)=	6.3	Y(20)=	3.775
X(21)=	66.6	Y(21)=	1.166
X(22)=	19.2	Y(22)=	.147
X(23)=	44.4	Y(23)=	.399
X(24)=	64.2	Y(24)=	-2.398
X(25)=	70.5	Y(25)=	-.565
X(26)=	84.0	Y(26)=	-.422
X(27)=	46.7	Y(27)=	-2.289
X(28)=	26.7	Y(28)=	7.514
X(29)=	93.7	Y(29)=	2.668
X(30)=	49.5	Y(30)=	.852
X(31)=	19.5	Y(31)=	3.747
X(32)=	46.6	Y(32)=	1.033
X(33)=	74.3	Y(33)=	4.388
X(34)=	7.1	Y(34)=	3.494
X(35)=	33.3	Y(35)=	2.031
X(36)=	4.2	Y(36)=	1.678
X(37)=	66.4	Y(37)=	2.115
X(38)=	85.5	Y(38)=	3.023
X(39)=	15.2	Y(39)=	.081
X(40)=	6.3	Y(40)=	2.970
X(41)=	66.6	Y(41)=	-.631
X(42)=	19.2	Y(42)=	.958
X(43)=	44.4	Y(43)=	3.417
X(44)=	64.2	Y(44)=	2.428
X(45)=	70.5	Y(45)=	1.668
X(46)=	84.0	Y(46)=	.819
X(47)=	46.7	Y(47)=	-.439
X(48)=	26.7	Y(48)=	1.572
X(49)=	93.7	Y(49)=	.534
X(50)=	49.5	Y(50)=	4.127
X(51)=	19.5	Y(51)=	4.030
X(52)=	46.6	Y(52)=	1.923
X(53)=	74.3	Y(53)=	1.487
X(54)=	7.1	Y(54)=	4.401
X(55)=	33.3	Y(55)=	.345
X(56)=	4.2	Y(56)=	1.000
X(57)=	66.4	Y(57)=	4.045
X(58)=	85.5	Y(58)=	3.161
X(59)=	15.2	Y(59)=	4.027
X(60)=	6.3	Y(60)=	6.324
X(61)=	66.6	Y(61)=	1.731
X(62)=	19.2	Y(62)=	3.928
X(63)=	44.4	Y(63)=	.596
X(64)=	64.2	Y(64)=	-2.585
X(65)=	70.5	Y(65)=	2.062
X(66)=	84.0	Y(66)=	2.221
X(67)=	46.7	Y(67)=	3.571
X(68)=	26.7	Y(68)=	3.790
X(69)=	93.7	Y(69)=	.560
X(70)=	49.5	Y(70)=	3.561
X(71)=	19.5	Y(71)=	4.472
X(72)=	46.6	Y(72)=	.973

```
X( 73)=   74.3    Y( 73)=          .745
X( 74)=    7.1    Y( 74)=         -.532
X( 75)=   33.3    Y( 75)=         2.819
X( 76)=    4.2    Y( 76)=         2.482
X( 77)=   66.4    Y( 77)=         4.136
X( 78)=   85.5    Y( 78)=         -.540
X( 79)=   15.2    Y( 79)=         2.133
X( 80)=    6.3    Y( 80)=         1.838
X( 81)=   66.6    Y( 81)=         4.253
X( 82)=   19.2    Y( 82)=         3.058
X( 83)=   44.4    Y( 83)=         -.065
X( 84)=   64.2    Y( 84)=         2.560
X( 85)=   70.5    Y( 85)=         2.617
X( 86)=   84.0    Y( 86)=         -.216
X( 87)=   46.7    Y( 87)=         5.741
X( 88)=   26.7    Y( 88)=         6.168
X( 89)=   93.7    Y( 89)=         2.747
X( 90)=   49.5    Y( 90)=         3.155
X( 91)=   19.5    Y( 91)=         -.925
X( 92)=   46.6    Y( 92)=         4.183
X( 93)=   74.3    Y( 93)=        -1.836
X( 94)=    7.1    Y( 94)=         4.693
X( 95)=   33.3    Y( 95)=         3.454
X( 96)=    4.2    Y( 96)=          .125
X( 97)=   66.4    Y( 97)=          .386
X( 98)=   85.5    Y( 98)=         5.918
X( 99)=   15.2    Y( 99)=         2.400
X(100)=    6.3    Y(100)=         1.512
T(A0)=       1.196 T(A1)=      -1.334
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=2) WITH NEGATIVE REGRESSION

```
A0=    2.0 EST A0=    1.482 A1=  -2.0 EST A1=  -1.995
X(  1)=   27.3    Y(  1)=       -49.664
X(  2)=   66.4    Y(  2)=      -130.451
X(  3)=   22.7    Y(  3)=       -43.967
X(  4)=   14.4    Y(  4)=       -26.933
X(  5)=   30.0    Y(  5)=       -56.668
X(  6)=   73.3    Y(  6)=      -145.093
X(  7)=   99.2    Y(  7)=      -196.328
X(  8)=   11.0    Y(  8)=       -21.292
X(  9)=   50.3    Y(  9)=       -99.506
X( 10)=   28.0    Y( 10)=       -51.090
X( 11)=   73.4    Y( 11)=      -142.963
X( 12)=   57.4    Y( 12)=      -115.245
X( 13)=   30.9    Y( 13)=       -58.058
X( 14)=   30.5    Y( 14)=       -62.920
X( 15)=   19.2    Y( 15)=       -36.151
X( 16)=   48.6    Y( 16)=       -97.272
X( 17)=   57.5    Y( 17)=      -114.203
```

```
X( 18)=  67.6   Y( 18)=   -132.663
X( 19)=   1.0   Y( 19)=       .826
X( 20)=  90.9   Y( 20)=   -178.453
X( 21)=  27.3   Y( 21)=    -52.822
X( 22)=  66.4   Y( 22)=   -127.000
X( 23)=  22.7   Y( 23)=    -40.309
X( 24)=  14.4   Y( 24)=    -27.466
X( 25)=  30.0   Y( 25)=    -57.794
X( 26)=  73.3   Y( 26)=   -143.211
X( 27)=  99.2   Y( 27)=   -195.838
X( 28)=  11.0   Y( 28)=    -20.594
X( 29)=  50.3   Y( 29)=    -97.000
X( 30)=  28.0   Y( 30)=    -55.175
X( 31)=  73.4   Y( 31)=   -146.041
X( 32)=  57.4   Y( 32)=   -113.715
X( 33)=  30.9   Y( 33)=    -60.319
X( 34)=  30.5   Y( 34)=    -63.373
X( 35)=  19.2   Y( 35)=    -35.197
X( 36)=  48.6   Y( 36)=    -95.310
X( 37)=  57.5   Y( 37)=   -113.633
X( 38)=  67.6   Y( 38)=   -133.885
X( 39)=   1.0   Y( 39)=     -2.587
X( 40)=  90.9   Y( 40)=   -178.458
X( 41)=  27.3   Y( 41)=    -51.819
X( 42)=  66.4   Y( 42)=   -131.390
X( 43)=  22.7   Y( 43)=    -46.490
X( 44)=  14.4   Y( 44)=    -27.840
X( 45)=  30.0   Y( 45)=    -58.760
X( 46)=  73.3   Y( 46)=   -143.169
X( 47)=  99.2   Y( 47)=   -199.188
X( 48)=  11.0   Y( 48)=    -19.736
X( 49)=  50.3   Y( 49)=   -100.334
X( 50)=  28.0   Y( 50)=    -59.101
X( 51)=  73.4   Y( 51)=   -146.958
X( 52)=  57.4   Y( 52)=   -110.025
X( 53)=  30.9   Y( 53)=    -62.421
X( 54)=  30.5   Y( 54)=    -59.667
X( 55)=  19.2   Y( 55)=    -36.083
X( 56)=  48.6   Y( 56)=    -93.188
X( 57)=  57.5   Y( 57)=   -110.902
X( 58)=  67.6   Y( 58)=   -132.947
X( 59)=   1.0   Y( 59)=       .158
X( 60)=  90.9   Y( 60)=   -182.304
X( 61)=  27.3   Y( 61)=    -50.657
X( 62)=  66.4   Y( 62)=   -131.620
X( 63)=  22.7   Y( 63)=    -46.512
X( 64)=  14.4   Y( 64)=    -28.054
X( 65)=  30.0   Y( 65)=    -57.565
X( 66)=  73.3   Y( 66)=   -142.967
X( 67)=  99.2   Y( 67)=   -196.377
X( 68)=  11.0   Y( 68)=    -18.718
X( 69)=  50.3   Y( 69)=   -101.507
X( 70)=  28.0   Y( 70)=    -54.867
```

```
X( 71)=  73.4   Y( 71)=    -143.716
X( 72)=  57.4   Y( 72)=    -110.175
X( 73)=  30.9   Y( 73)=     -62.363
X( 74)=  30.5   Y( 74)=     -59.801
X( 75)=  19.2   Y( 75)=     -36.808
X( 76)=  48.6   Y( 76)=     -94.905
X( 77)=  57.5   Y( 77)=    -114.012
X( 78)=  67.6   Y( 78)=    -131.848
X( 79)=   1.0   Y( 79)=      -2.934
X( 80)=  90.9   Y( 80)=    -181.990
X( 81)=  27.3   Y( 81)=     -51.335
X( 82)=  66.4   Y( 82)=    -137.690
X( 83)=  22.7   Y( 83)=     -44.374
X( 84)=  14.4   Y( 84)=     -28.108
X( 85)=  30.0   Y( 85)=     -60.211
X( 86)=  73.3   Y( 86)=    -144.604
X( 87)=  99.2   Y( 87)=    -197.407
X( 88)=  11.0   Y( 88)=     -19.539
X( 89)=  50.3   Y( 89)=     -96.521
X( 90)=  28.0   Y( 90)=     -52.473
X( 91)=  73.4   Y( 91)=    -144.314
X( 92)=  57.4   Y( 92)=    -112.164
X( 93)=  30.9   Y( 93)=     -58.145
X( 94)=  30.5   Y( 94)=     -59.775
X( 95)=  19.2   Y( 95)=     -35.374
X( 96)=  48.6   Y( 96)=     -96.463
X( 97)=  57.5   Y( 97)=    -112.962
X( 98)=  67.6   Y( 98)=    -134.590
X( 99)=   1.0   Y( 99)=      -1.868
X(100)=  90.9   Y(100)=    -179.516
T(A0)=      -1.435 T(A1)=       .741
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=4) WITH POSITIVE REGRESSION

```
A0=    4.0 EST A0=    3.703 A1=    2.0 EST A1=    2.000
X(  1)=  95.0   Y(  1)=     195.493
X(  2)=  68.8   Y(  2)=     140.000
X(  3)=  86.2   Y(  3)=     175.047
X(  4)=  83.5   Y(  4)=     168.595
X(  5)=  59.1   Y(  5)=     128.804
X(  6)=  11.4   Y(  6)=      27.834
X(  7)=  53.0   Y(  7)=     110.245
X(  8)=  33.0   Y(  8)=      69.596
X(  9)=  20.8   Y(  9)=      44.049
X( 10)=   4.2   Y( 10)=       8.562
X( 11)=  83.9   Y( 11)=     167.895
X( 12)=  93.6   Y( 12)=     188.810
X( 13)=  31.8   Y( 13)=      67.665
X( 14)=  55.3   Y( 14)=     109.422
X( 15)=  43.3   Y( 15)=      87.839
```

X(16)=	46.4	Y(16)=	95.477
X(17)=	86.5	Y(17)=	175.495
X(18)=	54.4	Y(18)=	112.855
X(19)=	63.4	Y(19)=	129.515
X(20)=	72.3	Y(20)=	150.436
X(21)=	95.0	Y(21)=	194.778
X(22)=	68.8	Y(22)=	144.500
X(23)=	86.2	Y(23)=	171.964
X(24)=	83.5	Y(24)=	177.128
X(25)=	59.1	Y(25)=	120.153
X(26)=	11.4	Y(26)=	37.199
X(27)=	53.0	Y(27)=	104.825
X(28)=	33.0	Y(28)=	68.593
X(29)=	20.8	Y(29)=	46.661
X(30)=	4.2	Y(30)=	9.990
X(31)=	83.9	Y(31)=	175.340
X(32)=	93.6	Y(32)=	193.471
X(33)=	31.8	Y(33)=	68.742
X(34)=	55.3	Y(34)=	114.114
X(35)=	43.3	Y(35)=	83.347
X(36)=	46.4	Y(36)=	97.001
X(37)=	86.5	Y(37)=	174.236
X(38)=	54.4	Y(38)=	112.011
X(39)=	63.4	Y(39)=	132.287
X(40)=	72.3	Y(40)=	156.024
X(41)=	95.0	Y(41)=	206.382
X(42)=	68.8	Y(42)=	133.321
X(43)=	86.2	Y(43)=	181.200
X(44)=	83.5	Y(44)=	165.980
X(45)=	59.1	Y(45)=	115.821
X(46)=	11.4	Y(46)=	22.883
X(47)=	53.0	Y(47)=	107.726
X(48)=	33.0	Y(48)=	67.909
X(49)=	20.8	Y(49)=	45.594
X(50)=	4.2	Y(50)=	19.739
X(51)=	83.9	Y(51)=	175.104
X(52)=	93.6	Y(52)=	198.451
X(53)=	31.8	Y(53)=	62.138
X(54)=	55.3	Y(54)=	111.127
X(55)=	43.3	Y(55)=	91.176
X(56)=	46.4	Y(56)=	106.846
X(57)=	86.5	Y(57)=	177.296
X(58)=	54.4	Y(58)=	111.488
X(59)=	63.4	Y(59)=	123.380
X(60)=	72.3	Y(60)=	145.933
X(61)=	95.0	Y(61)=	190.307
X(62)=	68.8	Y(62)=	150.461
X(63)=	86.2	Y(63)=	174.757
X(64)=	83.5	Y(64)=	179.153
X(65)=	59.1	Y(65)=	127.810
X(66)=	11.4	Y(66)=	28.888
X(67)=	53.0	Y(67)=	114.946
X(68)=	33.0	Y(68)=	63.546

```
X( 69)=  20.8   Y( 69)=      40.846
X( 70)=   4.2   Y( 70)=      13.807
X( 71)=  83.9   Y( 71)=     163.189
X( 72)=  93.6   Y( 72)=     187.751
X( 73)=  31.8   Y( 73)=      67.855
X( 74)=  55.3   Y( 74)=     116.459
X( 75)=  43.3   Y( 75)=      91.324
X( 76)=  46.4   Y( 76)=      93.010
X( 77)=  86.5   Y( 77)=     176.677
X( 78)=  54.4   Y( 78)=     111.284
X( 79)=  63.4   Y( 79)=     122.792
X( 80)=  72.3   Y( 80)=     144.161
X( 81)=  95.0   Y( 81)=     190.551
X( 82)=  68.8   Y( 82)=     143.922
X( 83)=  86.2   Y( 83)=     180.633
X( 84)=  83.5   Y( 84)=     164.645
X( 85)=  59.1   Y( 85)=     124.118
X( 86)=  11.4   Y( 86)=      31.212
X( 87)=  53.0   Y( 87)=     114.487
X( 88)=  33.0   Y( 88)=      71.502
X( 89)=  20.8   Y( 89)=      48.419
X( 90)=   4.2   Y( 90)=      12.196
X( 91)=  83.9   Y( 91)=     171.593
X( 92)=  93.6   Y( 92)=     189.372
X( 93)=  31.8   Y( 93)=      65.891
X( 94)=  55.3   Y( 94)=     110.112
X( 95)=  43.3   Y( 95)=      87.793
X( 96)=  46.4   Y( 96)=      95.494
X( 97)=  86.5   Y( 97)=     180.377
X( 98)=  54.4   Y( 98)=     107.401
X( 99)=  63.4   Y( 99)=     130.525
X(100)=  72.3   Y(100)=     142.710
T(A0)=      -.287 T(A1)=     -.019
```

```
COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=4) WITH HORIZONTAL REGRESSION
```

```
A0=   4.0 EST A0=   4.245 A1=   0.0 EST A1=    -.010
X(  1)=  66.6   Y(  1)=       1.116
X(  2)=  19.2   Y(  2)=       4.102
X(  3)=  44.4   Y(  3)=      -5.569
X(  4)=  64.2   Y(  4)=      11.458
X(  5)=  70.5   Y(  5)=      -1.452
X(  6)=  84.0   Y(  6)=       3.057
X(  7)=  46.7   Y(  7)=        .347
X(  8)=  26.7   Y(  8)=       1.779
X(  9)=  93.7   Y(  9)=       6.711
X( 10)=  49.5   Y( 10)=       2.504
X( 11)=  19.5   Y( 11)=       8.518
X( 12)=  46.6   Y( 12)=       8.112
X( 13)=  74.3   Y( 13)=       4.648
```

X(14)=	7.1	Y(14)=	9.484
X(15)=	33.3	Y(15)=	9.981
X(16)=	4.2	Y(16)=	1.499
X(17)=	66.4	Y(17)=	3.398
X(18)=	85.5	Y(18)=	3.097
X(19)=	15.2	Y(19)=	-.222
X(20)=	6.3	Y(20)=	4.978
X(21)=	66.6	Y(21)=	-1.999
X(22)=	19.2	Y(22)=	2.203
X(23)=	44.4	Y(23)=	8.946
X(24)=	64.2	Y(24)=	9.590
X(25)=	70.5	Y(25)=	7.495
X(26)=	84.0	Y(26)=	6.021
X(27)=	46.7	Y(27)=	8.528
X(28)=	26.7	Y(28)=	6.375
X(29)=	93.7	Y(29)=	10.923
X(30)=	49.5	Y(30)=	1.532
X(31)=	19.5	Y(31)=	5.562
X(32)=	46.6	Y(32)=	2.373
X(33)=	74.3	Y(33)=	-.675
X(34)=	7.1	Y(34)=	3.777
X(35)=	33.3	Y(35)=	3.090
X(36)=	4.2	Y(36)=	.623
X(37)=	66.4	Y(37)=	3.738
X(38)=	85.5	Y(38)=	3.793
X(39)=	15.2	Y(39)=	-3.849
X(40)=	6.3	Y(40)=	8.167
X(41)=	66.6	Y(41)=	7.205
X(42)=	19.2	Y(42)=	4.623
X(43)=	44.4	Y(43)=	-.216
X(44)=	64.2	Y(44)=	.043
X(45)=	70.5	Y(45)=	.764
X(46)=	84.0	Y(46)=	1.306
X(47)=	46.7	Y(47)=	9.028
X(48)=	26.7	Y(48)=	7.292
X(49)=	93.7	Y(49)=	3.456
X(50)=	49.5	Y(50)=	8.881
X(51)=	19.5	Y(51)=	10.927
X(52)=	46.6	Y(52)=	.953
X(53)=	74.3	Y(53)=	10.321
X(54)=	7.1	Y(54)=	6.389
X(55)=	33.3	Y(55)=	4.518
X(56)=	4.2	Y(56)=	.068
X(57)=	66.4	Y(57)=	4.399
X(58)=	85.5	Y(58)=	4.870
X(59)=	15.2	Y(59)=	4.842
X(60)=	6.3	Y(60)=	7.675
X(61)=	66.6	Y(61)=	4.729
X(62)=	19.2	Y(62)=	-.635
X(63)=	44.4	Y(63)=	2.939
X(64)=	64.2	Y(64)=	2.815
X(65)=	70.5	Y(65)=	-1.647
X(66)=	84.0	Y(66)=	4.910

```
X( 67)=  46.7   Y( 67)=      -2.150
X( 68)=  26.7   Y( 68)=        .528
X( 69)=  93.7   Y( 69)=       4.308
X( 70)=  49.5   Y( 70)=       4.549
X( 71)=  19.5   Y( 71)=        .611
X( 72)=  46.6   Y( 72)=       3.854
X( 73)=  74.3   Y( 73)=      -2.362
X( 74)=   7.1   Y( 74)=       1.322
X( 75)=  33.3   Y( 75)=       6.267
X( 76)=   4.2   Y( 76)=       3.832
X( 77)=  66.4   Y( 77)=      -2.620
X( 78)=  85.5   Y( 78)=       2.266
X( 79)=  15.2   Y( 79)=       1.855
X( 80)=   6.3   Y( 80)=       7.504
X( 81)=  66.6   Y( 81)=      -1.425
X( 82)=  19.2   Y( 82)=      10.424
X( 83)=  44.4   Y( 83)=      -1.583
X( 84)=  64.2   Y( 84)=      -2.091
X( 85)=  70.5   Y( 85)=       8.261
X( 86)=  84.0   Y( 86)=        .835
X( 87)=  46.7   Y( 87)=       2.989
X( 88)=  26.7   Y( 88)=      10.085
X( 89)=  93.7   Y( 89)=       1.481
X( 90)=  49.5   Y( 90)=        .538
X( 91)=  19.5   Y( 91)=       2.616
X( 92)=  46.6   Y( 92)=       7.074
X( 93)=  74.3   Y( 93)=       9.274
X( 94)=   7.1   Y( 94)=        .574
X( 95)=  33.3   Y( 95)=        .335
X( 96)=   4.2   Y( 96)=       7.917
X( 97)=  66.4   Y( 97)=       6.679
X( 98)=  85.5   Y( 98)=       -.016
X( 99)=  15.2   Y( 99)=      11.187
X(100)=   6.3   Y(100)=       -.347
T(A0)=        .321 T(A1)=     -.701
```

COMBINED SAMPLE ESTIMATES OF A0 AND A1 WITH T(A0) AND T(A1)
FOR NORMAL (VARIANCE=4) WITH NEGATIVE REGRESSION

```
A0=   4.0 EST A0=   3.918 A1=  -2.0 EST A1=  -1.990
X(  1)=  27.3   Y(  1)=     -49.861
X(  2)=  66.4   Y(  2)=    -134.994
X(  3)=  22.7   Y(  3)=     -39.187
X(  4)=  14.4   Y(  4)=     -23.479
X(  5)=  30.0   Y(  5)=     -49.510
X(  6)=  73.3   Y(  6)=    -145.520
X(  7)=  99.2   Y(  7)=    -189.949
X(  8)=  11.0   Y(  8)=     -22.038
X(  9)=  50.3   Y(  9)=     -93.626
X( 10)=  28.0   Y( 10)=     -51.153
X( 11)=  73.4   Y( 11)=    -137.859
```

```
X( 12)=  57.4   Y( 12)=   -108.184
X( 13)=  30.9   Y( 13)=    -60.569
X( 14)=  30.5   Y( 14)=    -56.853
X( 15)=  19.2   Y( 15)=    -31.675
X( 16)=  48.6   Y( 16)=    -88.878
X( 17)=  57.5   Y( 17)=   -114.699
X( 18)=  67.6   Y( 18)=   -129.180
X( 19)=   1.0   Y( 19)=     -1.159
X( 20)=  90.9   Y( 20)=   -177.678
X( 21)=  27.3   Y( 21)=    -47.377
X( 22)=  66.4   Y( 22)=   -127.294
X( 23)=  22.7   Y( 23)=    -39.070
X( 24)=  14.4   Y( 24)=    -19.746
X( 25)=  30.0   Y( 25)=    -54.961
X( 26)=  73.3   Y( 26)=   -144.955
X( 27)=  99.2   Y( 27)=   -184.169
X( 28)=  11.0   Y( 28)=    -15.841
X( 29)=  50.3   Y( 29)=    -91.813
X( 30)=  28.0   Y( 30)=    -58.524
X( 31)=  73.4   Y( 31)=   -143.214
X( 32)=  57.4   Y( 32)=   -112.324
X( 33)=  30.9   Y( 33)=    -60.292
X( 34)=  30.5   Y( 34)=    -60.960
X( 35)=  19.2   Y( 35)=    -36.967
X( 36)=  48.6   Y( 36)=    -92.153
X( 37)=  57.5   Y( 37)=   -104.759
X( 38)=  67.6   Y( 38)=   -126.823
X( 39)=   1.0   Y( 39)=     -3.187
X( 40)=  90.9   Y( 40)=   -180.890
X( 41)=  27.3   Y( 41)=    -56.572
X( 42)=  66.4   Y( 42)=   -127.273
X( 43)=  22.7   Y( 43)=    -38.634
X( 44)=  14.4   Y( 44)=    -23.694
X( 45)=  30.0   Y( 45)=    -60.093
X( 46)=  73.3   Y( 46)=   -144.071
X( 47)=  99.2   Y( 47)=   -194.068
X( 48)=  11.0   Y( 48)=    -13.325
X( 49)=  50.3   Y( 49)=    -89.681
X( 50)=  28.0   Y( 50)=    -53.576
X( 51)=  73.4   Y( 51)=   -144.250
X( 52)=  57.4   Y( 52)=   -112.143
X( 53)=  30.9   Y( 53)=    -59.696
X( 54)=  30.5   Y( 54)=    -60.748
X( 55)=  19.2   Y( 55)=    -33.939
X( 56)=  48.6   Y( 56)=    -95.109
X( 57)=  57.5   Y( 57)=   -110.498
X( 58)=  67.6   Y( 58)=   -132.147
X( 59)=   1.0   Y( 59)=      -.894
X( 60)=  90.9   Y( 60)=   -183.781
X( 61)=  27.3   Y( 61)=    -57.448
X( 62)=  66.4   Y( 62)=   -122.933
X( 63)=  22.7   Y( 63)=    -37.878
X( 64)=  14.4   Y( 64)=    -23.321
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X( 65)=  30.0   Y( 65)=     -56.904
X( 66)=  73.3   Y( 66)=    -146.866
X( 67)=  99.2   Y( 67)=    -195.648
X( 68)=  11.0   Y( 68)=     -14.488
X( 69)=  50.3   Y( 69)=    -103.228
X( 70)=  28.0   Y( 70)=     -48.307
X( 71)=  73.4   Y( 71)=    -140.965
X( 72)=  57.4   Y( 72)=    -107.643
X( 73)=  30.9   Y( 73)=     -62.779
X( 74)=  30.5   Y( 74)=     -56.215
X( 75)=  19.2   Y( 75)=     -30.590
X( 76)=  48.6   Y( 76)=     -89.744
X( 77)=  57.5   Y( 77)=    -111.918
X( 78)=  67.6   Y( 78)=    -125.150
X( 79)=   1.0   Y( 79)=      -2.282
X( 80)=  90.9   Y( 80)=    -174.353
X( 81)=  27.3   Y( 81)=     -50.003
X( 82)=  66.4   Y( 82)=    -122.272
X( 83)=  22.7   Y( 83)=     -40.801
X( 84)=  14.4   Y( 84)=     -22.629
X( 85)=  30.0   Y( 85)=     -53.396
X( 86)=  73.3   Y( 86)=    -145.342
X( 87)=  99.2   Y( 87)=    -196.907
X( 88)=  11.0   Y( 88)=      -7.332
X( 89)=  50.3   Y( 89)=     -96.456
X( 90)=  28.0   Y( 90)=     -54.719
X( 91)=  73.4   Y( 91)=    -145.361
X( 92)=  57.4   Y( 92)=    -110.822
X( 93)=  30.9   Y( 93)=     -57.543
X( 94)=  30.5   Y( 94)=     -59.363
X( 95)=  19.2   Y( 95)=     -34.922
X( 96)=  48.6   Y( 96)=     -92.060
X( 97)=  57.5   Y( 97)=    -113.017
X( 98)=  67.6   Y( 98)=    -125.834
X( 99)=   1.0   Y( 99)=        .650
X(100)=  90.9   Y(100)=    -176.604
T(A0)=       -.114 T(A1)=       .732
```

A STUDY OF THE EFFECT OF NON-NORMAL DISTRIBUTIONS UPON SIMPLE LINEAR REGRESSION

by

DANIEL JOSEPH WICHLAN

B. S., St. Louis University, 1964

AN ABSTRACT OF A MASTER'S THESIS

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Very little is known about the changes which occur in the value of the regression function when the error about the function is non-normally distributed. Consequently, the purpose of this paper was to investigate the effect on the regression function when the dependent variable is treated as a normally distributed variable when in fact it is not normally distributed. The study considered Model I of simple linear regression in which the independent variable is treated as an observable parameter.

Three gamma distributions with parameters corresponding to exponential, chi-square, and negatively skewed normal distributions respectively were used. Each distribution was simulated about a positive, horizontal, and negative regression function. Three control normal distributions with parameters equal to each of the experimental distributions were also simulated. A sample of size twenty was taken in each case and each sample was replicated five times. In addition, an overall estimate was obtained by combining the samples from the five replications. The estimates of the coefficients in the regression functions obtained from treating the non-normal variates as normal variates were tested for significant variation from the known parameters by appropriate t-tests at the 10%, 5%, and 1% levels of significance. These tests led to the conclusion that there was no significant variation in the coefficients of Model I for simple linear regression.

The principle result of the study is that the regression function of Model I which is computed from data assumed to be

normally distributed is a satisfactory representation of the true regression function when the data is in fact from a non-normal distribution. This conclusion does not necessarily include other types of regressions such as the non-linear or multiple classifications. In particular, this result cannot be extended to Model II, the model in which the independent variable is considered to be jointly distributed with the dependent variable. Significant differences between the regression equations for normal and non-normal regression analysis have been demonstrated for the case of Model II. Thus, further investigation is required before this result can be applied to other models.